

Department of Mathematics and Statistics, University of Helsinki  
Numerical methods and the C language, Winter and Spring 2016

Workshop 7

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1. Compute the value of the integral  $I_m = \int_0^1 \cos(\pi m x) dx$  with four different methods (a) The trapez formula, (b) Romberg integration (NR: :qromb or in GSL the method romberg at the [www](#)-page), (c) Gauss (NRC) or Gauss-Kronrod (GSL) method, (d) a method of your choice (but different from (a)-(c)). Compare the accuracy of the results.
2. On the [www](#)-page you will find a data file of xy-pairs. Find the value of  $\sigma$  that minimizes

$$g(\sigma) \equiv \sum_{k=0}^{m-1} (y[k] - f(\sigma, x[k]))^2,$$

where  $f(\sigma, x) = (x/\sigma) * \exp(-x^2/(2.0 * \sigma^2))$ .

*Hint:* Compute the value of  $g(\sigma)$  when  $\sigma = 0.1 : 0.1 : 10$  and choose the one that minimizes  $g(\sigma)$ .

3. Modify the program `mypolyarea.cpp` as to calculate the area of:

a) A polygonal domain with the shape of the letter C:

```
x  0  6  6  4  4  2  2  4  4  6  6  0  0
y  0  0  3  3  2  2  5  5  4  4  7  7  0
```

b) A polygonal domain with the shape of two triangles, where the polygonal line intersects itself:

```
x  0  1  1  -1  -1  0
y  0  -1  1  -1  1  0
```

4. Let  $A, B$  be  $m \times n$  and  $p \times q$  matrices. Their Kronecker product  $\text{Kron}(A, B)$  (also called tensor or direct product) is the  $mp \times nq$  matrix

$(a(1,1)B \dots a(1,n)B; \dots; a(m,1)B \dots a(m,n)B)$ ;

For simplicity we write here  $\text{Kron}(A, B) = [A, B]$ . The program `Kronecker` on the [www](#)-page gives an implementation of this product. Verify experimentally the following properties (under suitable hypotheses)

- (a)  $[A, [B, C]] = [[A, B], C]$ ,
- (b)  $[A, B + C] = [A, B] + [A, C]$ ,
- (c)  $[A^{-1}, B^{-1}] = [A, B]^{-1}$
- (d)  $[A^T, B^T] = [A, B]^T$
- (e) If  $B$  and  $C$  have SVDs  $B = U_1 W_1 V_1^T$ ,  $C = U_2 W_2 V_2^T$ , then  $[B, C] = [U_1, U_2][W_1, W_2]([V_1, V_2]^T)$ .
- (f)  $[A^+, B^+]^+ = [A, B]$ .

5. On the course www-page some data of  $x, y, z$  values are given. Find the coefficients  $a, b, c$  that for the model  $ax + by + c = z$  provides the best fit to this data.

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-9.8453630878801288e+00	9.0916121374310990e+00	9.1754800449942611e+00
9.7116816042511172e+00	-9.0877979244514364e+00	1.9372308402961265e+00
4.3915465727409098e+00	5.2584261238847052e+00	7.5309994339621626e+00
4.5212450132338544e+00	8.1883741813657682e+00	-8.2489441792708558e+00
7.3507759335221614e+00	7.1167275389268658e+00	3.8727680751461389e+00
-5.4234284048077779e+00	-7.5549827877222482e+00	6.1411724407883233e+00
-8.4860762015385909e+00	7.1556082168387292e+00	-4.3861598588461801e+00
5.1308149821734128e+00	2.8066688463122906e+00	-1.3337349991005543e-01
1.8982809744301630e+00	-9.5789346422902000e-01	-8.9231700072638542e+00
-4.5691388680455927e-02	3.0140608330322713e+00	-2.6558042283429786e+00
-1.7275130523962494e+00	-5.1248048036940421e+00	-8.0956313750220605e+00
-6.9364306223282730e+00	-6.3445755542929172e+00	-7.2347443817345169e+00
-9.2278480246792771e+00	6.6380236934116219e+00	-8.4944038272343594e+00
9.0830409056893746e+00	7.6467686135539639e+00	2.8016523005448524e-01
4.8023768210794673e+00	5.8940053712083049e+00	-6.9013514355296977e+00
-1.2993257824794044e+00	2.9652750645602470e+00	-4.7813563117670626e+00
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-8.1326615429169777e+00	3.1410090407081923e+00	-2.0937765911658186e+00