# Descriptive set theory

## Exercise 2

1. Clearly we may assume that if $x \in a$ then $x \neq \emptyset$. Then for every $x \in a$ choose a surjection $f_x : \omega \to x$ and also a surjection $g : \omega \to a$ (w.o.l.g. $a \neq \emptyset$).

Then $F : \omega \times \omega \to \bigcup a$, $F(n,m) = f_{g(n)}(m)$

is a surj..

2. E.g. Suppose $cf(\omega_1) = \omega$ and let $f : \omega \to \omega_1$ be such that $\bigcup rng(f) = \omega_1$

Then $\omega_1 = \bigcup_{n<\omega} f(n)$ and thus by Ex 1.

$\omega_1$ is countable. $\lightning$

3. For a contradiction suppose $F : \mathbb{R}^2 \to \mathbb{R}$ is a homeomorphism. Let $a, b, c \in \mathbb{R}^2$ be s.t. $F(a) = -1$, $F(b) = 0$ and $F(c) = 1$

Then there is a cont. $f : [0,1] \to \mathbb{R}^2$

s.t. $f(0) = a$ and $f(1) = c$ and $b \notin rng(f)$.

Then $h := F \circ f$ is a cont. function from $[0,1]$

to $\mathbb{R}$ s.t. $h(0) = -1$, $h(1) = 1$ and

$0 \notin rng(h)$ $\lightning$.

4. If it is then $\mathbb{B}^2$ is homeomorphic with $\mathbb{R}^2$ and so ~~[illegible]~~ $\mathbb{R}$ is hom. with $\mathbb{B}$ which is hom. with $\mathbb{B}^2$ which is hom. with $\mathbb{R}^2$, a contradiction with Ex 3.

5. As the proof of Lemma 2.2.

6. It is enough to show that for all $X \in \mathrm{Borel}(S')$ there is $Y \in \mathrm{Borel}(S)$ s.t. $X = S' \cap Y$.

This is an easy induction on the definition of $\mathrm{Borel}(S')$ (i.e. use Lemma 1.5.3)