GEOMETRY OF CURVES AND SURFACES

Homework 9 (for the week Apr 4 - Apr 8)

(1) (a) A surface is called *flat* if its Gaussian curvature is zero everywhere. Calculate the Gaussian and mean curvatures of the cylinder $x^2+y^2 = R^2$, $R > 0$, and show that the cylinder is flat but not minimal.

(b) Calculate the Gaussian and mean curvatures of the helicoid

$$\mathbf{x}(u, v) = (u\cos v, u\sin v, bv),$$

where $b \neq 0$.

(2) Calculate the Gaussian and mean curvatures of the saddle surface $z = xy$. (Use the patch $\mathbf{x}(u, v) = (u, v, uv)$.)

(3) Find the Gaussian and mean curvatures of the cone

$$\mathbf{x}(u, v) = (u, v, \sqrt{u^2 + v^2}),$$

where $(u, v) \neq (0, 0)$.

(4) Show that the following patches have the same Gaussian curvature:

$$\mathbf{x}(u, v) = (u\cos v, u\sin v, v) \text{ and } \mathbf{y}(u, v) = (u\cos v, u\sin v, \ln u).$$

(5) Let $\mathbf{x}(u, v)$ be a regular coordinate patch, and let $c > 0$. Then $\mathbf{y}(u, v) = c\mathbf{x}(u, v)$ is also a regular coordinate patch. Let K_x and K_y denote the Gaussian curvatures of the patches $\mathbf{x}$ and $\mathbf{y}$, respectively. Show that

$$K_y = \frac{1}{c^2} K_x.$$

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