# GEOMETRY OF CURVES AND SURFACES

Homework 7 (for the week Mar 14 - Mar 18)

(1) Compute the second fundamental form of the elliptic paraboloid
$$\mathbf{x}(u, v) = (u, v, u^2 + v^2).$$

(2) Compute the normal and geodesic curvature of the circle $\alpha(t) = (\cos t, \sin t, 1)$ on the paraboloid $S$, where $S$ is parametrized by $\mathbf{x}(u, v) = (u, v, u^2 + v^2)$.

(3) Compute the geodesic curvature of any (not necessarily great) circle on a sphere of radius $R$.

(4) Consider the saddle surface $S$: $z = x^2 - y^2$.
   (a) Find a unit normal $N$ for $S$ at the point $p = (0, 0, 0)$.
   (b) Let $v = (1, 0, 0)$ be a unit vector in $T_pS$. Let $P$ be the plane determined by the vectors $N$ and $v$. The intersection $S \cap P$ is a smooth curve $\gamma$. Parametrize $\gamma$ and find its curvature.

(5) Consider again the saddle surface $S$: $z = x^2 - y^2$. Again, let $p = (0, 0, 0)$.
   (a) Let $v = (0, 1, 0)$ be a unit vector in $T_pS$. Let $P$ be the plane determined by the vectors $N$ and $v$. The intersection $S \cap P$ is a smooth curve $\gamma$. Parametrize $\gamma$ and find its curvature.
   (b) As part a, but use the tangent vector $(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0)$ instead of $(0, 1, 0)$.

*Date*: March 8, 2016.