GEOMETRY OF CURVES AND SURFACES

Homework 5 (for the week Feb 22 - Feb 26)

(1) Calculate the first fundamental forms of the following surfaces:
   (a) $\mathbf{x}(u,v) = (\sinh u \sinh v, \sinh u \cosh v, \sinh u)$
   (b) $\mathbf{x}(u,v) = (u - v, u + v, u^2 + v^2)$
   (c) $\mathbf{x}(u,v) = (\cosh u, \sinh u, v)$
   (d) $\mathbf{x}(u,v) = (u, v, u^2 + v^2)$

(2) Parametrize the circle $(x-2)^2 + (y-2)^2 = 1$ in the xy-plane with

$$\alpha(u) = (2 + \cos u, 2 + \sin u).$$

   (a) Write down a parametrization for the torus obtained by rotating the circle about the x-axis.
   (b) Calculate the first fundamental form for the torus using the parametrization obtained in part a.

(3) Consider the surface defined by

$$z = \frac{2xy}{x^2 + y^2}, \quad (x,y) \neq (0,0).$$

The surface has a patch

$$(u,v) \mapsto (u, v, \frac{2uv}{u^2+v^2}).$$

Convert (u,v) to polar coordinates and show that the surface is ruled.

(4) Find two different rulings for the surface defined by $z = xy$.

(5) Let S be a level surface, i.e., assume

$$S = \{(x,y,z) \in \mathbb{R}^3 \mid f(x,y,z) = c\},$$

where $f\colon \mathbb{R}^3 \to \mathbb{R}$ is a smooth function and the gradient ∇f does not vanish at any point of S. Show that ∇f is perpendicular to the tangent plane at every point of S. (Hint: take $v \in T_pS$ and show $\nabla f \cdot v = 0$.)

Date: February 21, 2016.