GEOMETRY OF CURVES AND SURFACES

Homework 3 (for the week Feb 8 - Feb 12)

(1) Calculate the curvature and torsion of the hyperbolic helix
$$\alpha(t) = (\cosh t, \sinh t, t).$$

(2) Calculate the curvature and torsion of the helix
$$\alpha(t) = (t + \sqrt{3}\sin t, 2\cos t, \sqrt{3}t - \sin t).$$

(3) Show that the curve
$$\alpha(t) = (\frac{4}{5}\cos t, 1 - \sin t, -\frac{3}{5}\cos t)$$
is a circle, and find its centre, radius and the plane where it lies.

(4) Let $\alpha(t)$ be a smooth unit speed curve with positive curvature. Let T, N and B denote the unit tangent vector, the principal normal vector and the binormal vector of α, respectively. There is a vector (called *Darboux vector*) ω satisfying the equations
$$T' = \omega \times T, \quad N' = \omega \times N, \quad B' = \omega \times B.$$
Use the Frenet-Serret equations to write ω as a linear combination of T, N and B.

(5) Let α, T, N, B and ω be as in Exercise 4. Show that
$$T' \times T'' = k^2\omega,$$
where k is the curvature of α.

Date: February 2, 2016.