GEOMETRY OF CURVES AND SURFACES

Homework 2 (for the week Feb 1 - Feb 5)

(1) Let
$$\alpha(t) = \left(\frac{(1+t)^{\frac{3}{2}}}{3}, \frac{(1-t)^{\frac{3}{2}}}{3}, \frac{t}{\sqrt{2}}\right), \quad -1 < t < 1.$$
   (a) Find the Frenet frame $\{T, N, B\}$ for α.
   (b) Calculate the curvature and torsion of α.

(2) Let
$$\alpha(t) = \left(\frac{1}{\sqrt{2}}\cos t, \sin t, \frac{1}{\sqrt{2}}\cos t\right).$$
   (a) Find the Frenet frame $\{T, N, B\}$ for α.
   (b) Calculate the curvature and torsion of α.

(3) Calculate the torsion of
$$\alpha\colon [1, \infty) \to \mathbb{R}^3, \quad t \mapsto \frac{1}{2}\left(t, \frac{1}{t}, \sqrt{2}\ln t\right).$$

(4) Let $a > 0$. Calculate the curvature and torsion of the helix
$$\alpha(t) = \left(a\cos\left(\frac{t}{c}\right), a\sin\left(\frac{t}{c}\right), \frac{bt}{c}\right),$$
   where $c = \sqrt{a^2 + b^2}$.

(5) Calculate T, N, B and the curvature and torsion for the curve
$$\alpha(t) = (e^t \cos t, e^t \sin t, e^t).$$