GEOMETRY OF CURVES AND SURFACES

Homework 12 (for the week May 2 - May 5)

(1) Let $\mathbb{H}$ denote the hyperbolic plane, and let $d_{\mathbb{H}}$ denote the hyperbolic distance. A formula for $d_{\mathbb{H}}(z, w)$ was proved in class. Show that

$$d_{\mathbb{H}}(z, w) = \ln \frac{|z - \bar{w}| + |z - w|}{|z - \bar{w}| - |z - w|}.$$

Here $z, w \in \mathbb{H}$, and $\bar{z}$ and $\bar{w}$ denote the complex conjugates of z and w, respectively.

(2) Calculate the hyperbolic distance between each pair of the four points

$$A = i, \;\; B = 1 + 2i, \;\; C = -1 + 2i, \;\; D = 7i.$$

(3) Determine whether or not there exists a positive real number s such that

$$d_{\mathbb{H}}(-s + i, i) = d_{\mathbb{H}}(i, s + i) = d_{\mathbb{H}}(-s + i, s + i).$$

(Here i denotes the imaginary unit.)

(4) Show that for any $a \in \mathbb{H}$ there is a unique hyperbolic line passing through a and intersecting the imaginary axis perpendicularly.

(5) Let l be the semicircle in $\mathbb{H}$ whose euclidean center is -2 and whose euclidean radius is 1. Then l is a hyperbolic line in $\mathbb{H}$. Find two hyperbolic lines through i that are parallel to l.

Date: April 28, 2016.