GEOMETRY OF CURVES AND SURFACES

Homework 11 (for the week Apr 18 - Apr 22)

(1) Show that if a surface patch has first fundamental form $e^\lambda(du^2 + dv^2)$, where λ is a smooth function of u and v, its Gaussian curvature K satisfies

$$\triangle\lambda + 2Ke^\lambda = 0,$$

where $\triangle$ denotes the Laplacian $\partial^2/\partial u^2 + \partial^2/\partial v^2$.

(2) Show that there is no surface patch whose first and second fundamental forms are

$$du^2 + (\cos^2 u)dv^2 \text{ and } (\cos^2 u)du^2 + dv^2,$$

respectively.

(3) Suppose that the first and second fundamental forms of a surface patch are $Edu^2 + Gdv^2$ and $Ldu^2 + Ndv^2$, respectively. Show that the Codazzi-Mainardi equations reduce to

$$L_v = \frac{1}{2}E_v\Big(\frac{L}{E} + \frac{N}{G}\Big), \quad N_u = \frac{1}{2}G_u\Big(\frac{L}{E} + \frac{N}{G}\Big).$$

Deduce that the principal curvatures $k_1 = L/E$ and $k_2 = N/G$ satisfy the equations

$$(k_1)_v = \frac{E_v}{2E}(k_2 - k_1), \quad (k_2)_u = \frac{G_u}{2G}(k_1 - k_2).$$

(4) Calculate the curvature of a sphere of radius R using the formula in Corollary 23.6 of the lecture notes (posted on the class homepage).

(5) Have a look at the proof of the Gauss equations (Prop. 23.4). Collect the coefficients of x_v in $(x_{uu})_v = (x_{uv})_u$, and use that to prove one of the Gauss equations.

Date: April 15, 2016.