GEOMETRY OF CURVES AND SURFACES

Homework 1 (for the week Jan 25 - Jan 29)

(1) Find the length of the parametric curve

$$\alpha(t) = (\ln(t), \frac{1}{2}t^2, \sqrt{2}\ t)$$

where $1 \leq t \leq 2$.

(2) Reparametrize the curve α with respect to arc length, where

$$\alpha\colon [1, \infty) \to \mathbb{R}^3, \ \ t \mapsto \frac{1}{2}(t, \frac{1}{t}, \sqrt{2}\ln t)$$

(3) Find the curvature of the following parametric curves:

(a)

$$\alpha\colon [1, \infty) \to \mathbb{R}^3, \ \ t \mapsto \frac{1}{2}(t, \frac{1}{t}, \sqrt{2}\ln t)$$

(b)

$$\alpha\colon \mathbb{R} \to \mathbb{R}^3, \ \ t \mapsto (\frac{R}{2}\cos\frac{\sqrt{2}}{R}t, \frac{R}{2}\sin\frac{\sqrt{2}}{R}t, \frac{t}{\sqrt{2}})$$

(Here $R > 0$.)

(4) Find the curvature of the following parametric curves at points where $\alpha'(t) \neq 0$:

(a) $\alpha(t) = (t, \cosh t)$

(b) $\alpha(t) = (\cos^3 t, \sin^3 t)$

(c) $\alpha(t) = (\frac{4}{5}\cos t, 1 - \sin t, -\frac{3}{5}\cos t)$

(5) Consider the parametric curve

$$\alpha(t) = (\sqrt{\pi}\int_0^t \cos(\frac{\pi u^2}{2})du, \sqrt{\pi}\int_0^t \sin(\frac{\pi u^2}{2}du)).$$

(a) Calculate the curvature $k(t)$ of α.

(b) Show that the length of α from the origin to $\alpha(t)$ equals $k(t)$, for every $t > 0$.

Date: January 20, 2016.