Evolution of Resource Use

(Adaptive Dynamics Project 2016)

Consider a consumer species exploiting two different types of resources with respective densities R_1 and R_2. The evolving trait of the species is the proportion x of time spent searching for resource of type 1. The proportion of time spent searching for the other resource thus is $1 - x$. The perfect generalist has a trait value $x = \frac{1}{2}$, whereas a perfect specialist has either $x = 0$ or $x = 1$. The aim of the project is to study resource specialisation via the adaptive dynamics of x.

Consider a resident population with trait values $x_1, \ldots, x_k$ and corresponding population densities $n_1, \ldots, n_k$, and let the population dynamics be given by

$$\dot{n}_i = n_i\, f(x_i \gamma_1 \beta_1 R_1, (1 - x_i)\gamma_2 \beta_2 R_2) - \mu n_i \quad (i = 1, \ldots, k),$$

$$\dot{R}_1 = a_1 - b_1 R_1 - \beta_1 R_1 \sum_{i=1}^{k} x_i n_i,$$

$$\dot{R}_2 = a_2 - b_2 R_2 - \beta_2 R_2 \sum_{i=1}^{k} (1 - x_i) n_i.$$

Assume that the resource a_j, b_j and β_j are large and the γ_j for $j = 1, 2$ are small compared to all other parameters, so that the R_j-dynamics become fast compared to the n_i-dynamics and we can use the quasi-equilibria of the resources in the equations for the n_i. This gives

$$R_1 = \frac{a_1}{b_1 + \beta_1 \sum_{i=1}^{k} x_i n_i},$$

and

$$R_2 = \frac{a_2}{b_2 + \beta_2 \sum_{i=1}^{k} (1 - x_i) n_i},$$

which upon substitution into the equation for n_i gives

$$\dot{n}_i = n_i\, f\left(\frac{x_i \gamma_1 \beta_1 a_1}{b_1 + \beta_1 \sum_{i=1}^{k} x_i n_i}, \frac{(1 - x_i)\gamma_2 \beta_2 a_2}{b_2 + \beta_2 \sum_{i=1}^{k} (1 - x_i) n_i} \right) - \mu n_i.$$

Study the evolution of $x \in [0, 1]$ if f is given by

$$f(p, q) = (p^\alpha + q^\alpha)^{\frac{1}{\alpha}}.$$

Different values of α lead to the following classification:

Parameter range:	**Resource types:**
$\alpha > 1$	Antagonistic
$\alpha >=$	Perfectly substitutable
$0 < \alpha < 1$	Complementary
$-\infty < \alpha < 0$	Hemi-essential
$\alpha = -\infty$	Essential

To get an idea of the meaning of these terms, you may want to plot the contours of the function f for different values of α.