Evolution of resistance in a host-pathogen system

(Adaptive Dynamics Project 2016)

In this project, we consider a pathogen that causes lifelong infection of its hosts, and assume that hosts may develop resistance against this pathogen (i.e., are able to defend against getting infected) but only at some cost. The question is how much resistance the host should evolve.[1] A basic ecological model of a lifelong infection is given by

$$\begin{aligned} \dot{S} &= r(\beta)\, N - q(N)\, S - \beta\, S\, I \\ \dot{I} &= \beta\, S\, I - \alpha\, I - q(N)\, I \end{aligned}$$

where S and I are respectively the densities of susceptible hosts and the number of infected hosts, and $N = S + I$ is the total population density. It is assumed that susceptible and infected individuals reproduce equally and newborns are free of infection, hence all the newborns are added to the susceptibles. The birth rate is an increasing function of the transmission rate β. The reason is that β varies inversely with the resistance of the host, and resistance is costly. The death rate, $q(N)$, depends linearly on total population density, so that in absence of the pathogen the host grows logistically. The virulence (i.e., infection induced death rate) is denoted by α.

Consider a population of hosts with different degrees of resistance corresponding to different values of the transmission rate. We thus have $\beta_1, \ldots, \beta_k$ and population densities $S_1, \ldots, S_k$. The population dynamics then become

$$\begin{aligned} \dot{S}_i &= r(\beta_i)\, N_i - S_i\, q\left(\textstyle\sum_{j=1}^{k} N_j\right) - \beta_i\, S_i \left(\textstyle\sum_{j=1}^{k} I_j\right) \\ \dot{I}_i &= \beta_i\, S_i \left(\textstyle\sum_{j=1}^{k} I_j\right) - \alpha\, I_i - I_i\, q\left(\textstyle\sum_{j=1}^{k} N_j\right) \end{aligned}$$

[1] The following background will hopefully further motivate this project but its understanding is not necessary for the immediate job. From basic models of epidemiology, such as the SIR model, you may be aware of the phenomenon of herd immunity. Herd immunity means that if a large enough fraction of the population is resistant or immune to the pathogen, then there is no epidemic outbreak, because the expected number of secondary infections during the lifetime of a primary infection (i.e. till recovery or death of the infected host) is less than 1 and is thus not enough to replace the primary infected host. The point here is that not every individual need to be resistant for herd immunity, and it is possible to enjoy protection from herd immunity without being resistant to the disease. When resistance is costly, there is an incentive to rely on herd immunity and avoid paying the cost. This is a form of cheating, because such non-resistant individuals enjoy benefit from others' costly resistance. Of course if non-resistant cheaters are too frequent, then herd immunity collapses. Yet we may expect the evolution of resistant and cheating strategies, and this is what the present project will elaborate on.

where $N_i = S_i + I_i$. To study the adaptive dynamics of host resistance it is probably easier to use the basic reproduction number R_0 as a fitness proxy than the invasion fitness itself.