Evolution of Mutualism

(Adaptive Dynamics Project 2016)

Mutualism is the phenomenon of two different species coexisting and interacting in a mutually beneficial manner (such as flowers and bees). Suppose two mutualistic species help each other best if their respective trait values match (like the morphology of the flower and the size of the bee). The traits are, however, also involved in adaptation to resource use, and the resources of the two species are best utilised with different trait values. There is thus a conflict between resource use (which is best if the two species have different trait values) and mutualism (which is maximal when the two trait values match).

Consider two species A and B that are potentially mutualistic. Although their traits may be of a totally different nature (like flower morphology and bee size), we assume that they can be parameterised by scalar valued x^{A} for species A and x^{B} for species B such that there is a perfect match between the two traits if and only if $x^{\mathrm{A}} = x^{\mathrm{B}}$. We will identify the parameterisations with the trait values so that we can talk about the evolution of x^{A} and x^{B}.

Consider a population of species A with trait values $x_1^{\mathrm{A}}, \ldots, x_{k_{\mathrm{A}}}^{\mathrm{A}}$ and corresponding population densities $n_1^{\mathrm{A}}, \ldots, n_{k_{\mathrm{A}}}^{\mathrm{A}}$, and likewise a population of species B with trait values $x_1^{\mathrm{B}}, \ldots, x_{k_{\mathrm{B}}}^{\mathrm{B}}$ and corresponding population densities $n_1^{\mathrm{B}}, \ldots, n_{k_{\mathrm{B}}}^{\mathrm{B}}$. Competition is intra-specific but not intra-specific (i.e., there is competition within species A and within species B but not between the two species), because they exploit different resources. We assume that in isolation from one another (i.e., without mutualistic benefits) both species grow logistically according to

$$\dot{n}_i^{\mathrm{A}} = r_{\mathrm{A}} n_i^{\mathrm{A}} \left(1 - \frac{\sum_{i'=1}^{k_{\mathrm{A}}} n_{i'}^{\mathrm{A}}}{K_{\mathrm{A}}(x_i^{\mathrm{A}})} \right) \quad (i = 1, \ldots, k_{\mathrm{A}}),$$

and

$$\dot{n}_j^{\mathrm{B}} = r_{\mathrm{B}} n_j^{\mathrm{B}} \left(1 - \frac{\sum_{j'=1}^{k_{\mathrm{B}}} n_{j'}^{\mathrm{B}}}{K_{\mathrm{B}}(x_j^{\mathrm{B}})} \right) \quad (j = 1, \ldots, k_{\mathrm{B}}).$$

Thus, in isolation from one another other, evolution will maximise the carrying capacities K_{A} and K_{A}. Show this! If both species occur together, however, there joined dynamics is given by

$$\dot{n}_i^{\mathrm{A}} = r_{\mathrm{A}}\, n_i^{\mathrm{A}} \left(1 - \frac{\sum_{i'=1}^{k_{\mathrm{A}}} n_{i'}^{\mathrm{A}}}{K_{\mathrm{A}}(x_i^{\mathrm{A}})} + \sum_{j'=1}^{k_{\mathrm{B}}} a_{\mathrm{AB}}(x_i^{\mathrm{A}} - x_{j'}^{\mathrm{B}}) n_{j'}^{\mathrm{A}} \right) \quad (i = 1, \dots, k_{\mathrm{A}}),$$

$$\dot{n}_j^{\mathrm{B}} = r_{\mathrm{B}}\, n_j^{\mathrm{B}} \left(1 - \frac{\sum_{j'=1}^{k_{\mathrm{B}}} n_{j'}^{\mathrm{B}}}{K_{\mathrm{B}}(x_j^{\mathrm{B}})} + \sum_{i'=1}^{k_{\mathrm{A}}} a_{\mathrm{B}}(x_j^{\mathrm{B}} - x_{i'}^{\mathrm{A}}) n_{i'}^{\mathrm{AB}} \right) \quad (j = 1, \dots, k_{\mathrm{B}}).$$

where the positive functions a_{AB} and a_{BA} describe the mutualistic effects of, respectively, species B on species A and *vice versa*.

Study the adaptive dynamics of mutualism if

$$K_{\mathrm{A}}(x) = e^{-(x - x_{\mathrm{opt}}^{\mathrm{A}})},$$

$$K_{\mathrm{B}}(x) = e^{-(x - x_{\mathrm{opt}}^{\mathrm{B}})},$$

and

$$a_{\mathrm{AB}}(\Delta x) = c_{\mathrm{AB}}\, e^{-\alpha_{\mathrm{AB}} \Delta x^2},$$

$$a_{\mathrm{BA}}(\Delta x) = c_{\mathrm{BA}}\, e^{-\alpha_{\mathrm{BA}} \Delta x^2}.$$

Note that that the two species may not equally benefit from their interaction.