Evolution of Dormancy

(Adaptive Dynamics Project 2016)

A seed in the soil is said to be dormant if it is alive but not actively growing. The collection of dormant seeds is called the seed bank. Dormancy is advantageous in temporally stochastic environments. As an extreme example, consider population of an annual plant in an environment where some years are so bad that the crop fails entirely. Without seed bank the population goes extinct the first time such a year comes about. However, if some of the seeds had been dormant, they would not have germinated in the fateful year but a later and possible better year, and hence the population would not have died out. In this project, we study the evolution of dormancy (or, equivalently, of seed germination) and in particular, the diversification of germination strategies in annual plants. The evolving trait is the annual germination probability $x \in [0, 1]$ per seed.

Consider a resident population of strategies $x_1, \ldots, x_k$ and corresponding seed densities $n_1, \ldots, n_k$ before germination. Suppose that a seed that germinates in year t produces

$$\frac{b}{a + \sum_{j=1}^{k} x_j n_j(t)}$$

new seeds if the year is good, but no seeds at all if the year is bad. Suppose further that good year happen with probability $p \in (0, 1)$ and that the type of successive years is stochastically independent of one another. The resident population dynamics then becomes

$$n_i(t+1) = \begin{cases} \sigma(1 - x_i)n_i(t) + \frac{b x_i n_i(t)}{a + \sum_{j=1}^{k} x_j n_j(t)} & \text{with probability } p \\ \\ \sigma(1 - x_i)n_i(t) & \text{with probability } 1 - p \end{cases}$$

where $\sigma \in (0, 1)$ is the seed survival probability in the seed bank. Study the adaptive dynamics of the germination probability in the above model. This can be done entirely analytically, but it would be nice to have some numerical examples as well.