Evolution of Cooperation

(Adaptive Dynamics Project 2016)

In this project, we investigate a model for the evolution of cooperative behaviour. The obvious problem with cooperation is that it is vulnerable to cheating. Selfish individuals ("defectors", who take the benefit offered by others but do not return the favour to others) are at an advantage in a population of cooperators. Many models of game theory address the joint dynamics of cooperators and defectors and investigate which mechanisms can protect a population of cooperators from the invasion of defectors. In this project, however, we are concerned with the evolution of the very strategies "cooperator" and "defector" themselves.

To that end, let the degree of investment in cooperation be parameterised by the variable x such that $x = 0$ corresponds to zero investment and $x = 1$ to maximal investment. For simplicity, we consider only interactions between randomly paired individuals. Both individuals get the same benefit (which is an increasing function of total investment), but they may pay a different cost (which is an increasing function of their individual investment). The pairing of individuals happens only once a year.

Consider a resident population of a species with strategies $x_1, \ldots, x_k$ and corresponding relative frequencies $p_1, \ldots, p_k$. The payoff (=benefit minus cost) to x_i against x_j is

$$\pi_{ij} = B(x_i + x_j) - C(x_i),$$

which corresponds to a *per capita* number $W(\pi_{ij})$ of offspring. Thus, with random pairing, the expected *per capita* number of offspring to x_i is

$$\sum_{j=1}^{k} W(\pi_{ij}) p_j(t),$$

and so the the relative frequency of x_i in the next year becomes

$$p_i(t+1) = \frac{\sigma p_i(t) + p_i(t) \sum_{j=1}^{k} W(\pi_{ij}) p_j(t)}{\sigma + \sum_{i'=1}^{k} \sum_{j'=1}^{k} W(\pi_{i'j'}) p_{i'}(t) p_{j'}(t)},$$

where $0 \leq \sigma < 1$ is the annual survival probability, and where the denominator is merely for normalisation so that the p_i sum up to one.

Study the adaptive dynamics of $x \in [0, 1]$ if $W(\pi) = \max\{0, a + \pi\}$ for $a \geq 0$.