# Evolution of Cannibalism – II

(Adaptive Dynamics Project 2016)

In some fish species the juveniles feed on small prey living in the mud on the bottom of the lake. The adults feed on the same prey but also on the juveniles of their own species. Optimal feeding on either prey, however, needs different behavioural or morphological adaptations. Suppose that these adaptations can be parameterised by a scalar variable $x \in [0,1]$ where $x = 1$ corresponds full specialisation of the adult on small prey living in the mud on the bottom of the lake, while $x = 0$ corresponds to full specialisation on attacking conspecific juveniles.

Consider a resident population of strategies $x_1, \ldots, x_k$ with corresponding population densities $u_1, \ldots, u_k$ of the juveniles and $v_1, \ldots, v_k$ of the adults, and let the population dynamics be given by

$$\dot{u}_i = \gamma_1 \beta_1(x_i) R v_i + \gamma_2 \beta_2(x_i) v_i \sum_{j=1}^{k} u_i - u_i \sum_{j=1}^{k} \beta_2(x_j) v_j - \delta u_i - \varepsilon u_i$$

$$\dot{v}_i = \varepsilon u_i - \zeta v_i$$

and for the small prey living in the mud on the bottom of the lake,

$$\dot{R} = rR\left(1 - \frac{R}{K}\right) - \beta_0 R \sum_{j=1}^{k} u_j - R \sum_{j=1}^{k} \beta(x_j) v_j.$$

Make sure you understand the model. For the attack rates $\beta_1$ and $\beta_2$ are traded-off against one another by

$$\begin{aligned} \beta_1(x) &= b_1 x^p \\ \beta_2(x) &= b_2 (1-x)^q \end{aligned}$$

for $p, q > 0$. To see the effect of the $p$ and the $q$, make a parametric plot of $\beta_2$ versus $\beta_1$ for $x \in [0,1]$, and study the adaptive dynamics of $x$.

Hint: this is a structured population with juveniles and adults. Instead of calculating the invasion fitness it is easier to use fitness proxies, of which the basic reproduction ratio is the most straight-forward.