Evolutionary Arms Race

(Adaptive Dynamics Project 2016)

Consider a trait such as body size, which influences success in competitive contests with other members of the population. The evolution of such a trait is determined by two opposing forces: Being larger than the opponents is advantageous for winning the contests, thereby for obtaining resources and achieving high fecundity. On the other hand, large size entails some opponent-independent cost either in terms of resources used for maintenance or in reducing the chance for survival. In this project, we explore the evolution of body size (or antler size or a similar trait) in the Lotka-Volterra competition model.

Let $x \in \mathbb{R}$ denote log-body size, and consider a resident population with trait values $x_1, \ldots, x_k$ and corresponding population densities $n_1, \ldots, n_k$. Suppose further that the population dynamics is given by

$$\dot{n}_i = n_i \left(r(x_i) - \sum_{j=1}^{k} a(x_i - x_j) n_j \right)$$

where $r(x) = b(x) - d(x)$ is the difference between the *per capita* birth rate $b(x)$ and (competition unrelated) death rate $d(x)$, and which therefore may take positive as well as negative values depending on whether birth or death dominates.

The competition kernel $a(x_i - x_j)$ describes interference competition by intimidation contests or real fights and is always positive. Its argument is the difference in log-body size (and hence the ratio of the true body sizes) of the two contestants. Assuming that a larger body size is always competitively advantageous, a is a decreasing function.

To explore the adaptive dynamics of body size x, start by investigating the monomorphic evolutionary singularities. To some extent this can be done analytically. For numerical work, take

$$a(\Delta x) = \gamma \left(1 - \frac{1}{1 + \nu e^{-k\Delta x}}\right),$$

which has sigmoidal shape and saturates for $x \to \pm\infty$: if the difference in size is big, the larger contestant will almost certainly win any fights, and hence increasing the size difference further has hardly an effect on the outcome of competition. Here are some suggestions of situations you may want to explore:

(i) Take $r(x) = \alpha - \beta x$ and $a(\Delta x)$ as above. Try to find a branching point and investigate the effects of the parameters.

(ii) Take $r(x) = b\left(\sqrt{x^2 + d} - x\right) - \beta$, a convex decreasing function with parameters $b = 10$, $d = 3.5$, $\beta = .06$ and take $a(\Delta x)$ as above with $\gamma = 2$, $\nu = .7$, and $k = .24$. Show that there are several monomorphic evolutionary singularities, but eventually the population will settle on a monomorphic ESS even if it first becomes dimorphic via evolutionary branching.

(iii) Assume $r(x) = \alpha - \beta x$ and $a(\Delta x)$ as above. Construct an example such that an initially monomorphic population stays monomorphic, but an initially dimorphic population can evolve higher levels of polymorphism.