ADAPTIVE DYNAMICS
EXERCISE 8

Exercise 8:
We study the evolution of virulence in the following SI model:

$$\begin{cases} \frac{dS}{dt} = \lambda S - \mu(N)S - S\sum_{j=1}^{k}\beta(\alpha_j)I_j \\ \\ \frac{dI_i}{dt} = S\beta(\alpha_i)I_i - \alpha_i I_i - \mu(N)I_i \end{cases}$$

where S denotes the density of susceptible individuals, I_i the density of infected individuals with strain $i = 1, \ldots, k$, and N the total population density. Consult exercise 2 before you continue. Next, take $\lambda = 2$, $\mu(N) = N$ and

$$\beta(\alpha) = 3 + \frac{2\alpha^2}{1 + .2\alpha}.$$

and answer the following questions.

(a) Determine the range of viable values of the virulence α, i.e., for which values the virus can invade the virgin environment.

(b) Make a pairwise invadability plot (PIP) over the the range $0 \leq \alpha \leq 2$. What kind of singular strategies are there in terms of their invadability and their dynamics stability?

(c) If you can, calculate the numerical values of each singular strategy using a computer, and evaluate the curvatures $\partial_y^2 s_x(y)$ and $\partial_x^2 s_x(y)$ at each singular strategy to see how they fit in the classification of singular strategies, i.e., the so-called "eight cases".

(d) Write down the canonical equation (CE) for the evolution of the virulence in a monomorphic population. Solve the CE numerically for different starting values.

(e) Give a mutual invadability plot (MIP) together with the evolutionary isoclines. What can you say about the long-term evolution of the dimorphic population?

(f) Write down the canonical equation (CE) for the evolution of the virulence in a monomorphic population. Solve the CE numerically for relevant starting values.

(g) If you want, play around with different parameter values and even a different form of the transmission rate $\beta\alpha$.