# ADAPTIVE DYNAMICS
# EXERCISE 6 – 7

**Exercise 6:**
We study the evolution of the predator's attack rate $x$ in the following prey-predator model:

$$\begin{cases} \dot{N} = N\Big(aN\left(1-\frac{N}{K}\right) - d - \sum_j x_j P_j\Big) \\ \\ \dot{P}_i = P_i\Big(\gamma x_i N - \delta\Big) \qquad \text{for all i} \end{cases}$$

**(a)** Interpret the model in terms of individual-level behaviour.

**(b)** Study the prey dynamics if there are no predators around. Note that the prey exhibits an Allee-effect, i.e., a negative growth rate when the population density is low. Why is this (in biological terms)?

**(c)** Study the prey-predator dynamics with only a single predator type present in order to convince yourself of the possibility of cycles. What do the dynamics look like for a very high predator attack rate $x$?

**(d)** Rewrite the model in a form with an explicit environmental feedback loop. Give the invasion fitness $s_E(y)$ and determine the essential dimension of the environment.

**(e)** Show that evolution of the predator's attack rate $x$ inevitably leads to the catastrophic extinction of both the prey and the predator.

**Exercise 7:**
Modify the prey-predator dynamics by introducing a Holling type-II functional response:

$$\begin{cases} \dot{N} = N\left(aN\left(1-\frac{N}{K}\right) - d - \sum_j \frac{x_j P_j}{1+hx_jN}\right) \\ \\ \dot{P}_i = P_i\left(\frac{\gamma x_j N}{1+hx_jN} - \delta\right) \qquad \text{for all i} \end{cases}$$

**(a)** Rewrite the model in a form with an explicit environmental feedback loop. Give the invasion fitness $s_E(y)$ and show that now the environment is essentially infinite-dimensional.

**(b)** Use the principle of selective neutrality of residents as well as a monotony argument to show that coexistence of different predator types is not possible (i.e., in spite of the environment being essentially infinitely-dimensional).

**(c)** Calculate the selection gradient and use this to argue that evolution of the predator's attack rate $x$ again leads to the catastrophic extinction of both the prey and the predator, i.e., at least if mutation steps are sufficiently small.