ADAPTIVE DYNAMICS
EXERCISE 1 – 4

Introduction:

Each of the following exercises gives the population dynamics of a k-morphic resident population with strategies $x_1, \ldots, x_k \in \mathbb{X}$ and corresponding population densities $n_1, \ldots, n_k \in \mathbb{R}_+$. For each of these exercises you are asked to do the following:

(a) Give an ecological interpretation of the model.

(b) Rewrite the dynamics in the form

$$\begin{cases} \dot{n}_i = r(x_i, E)n_i \quad \forall i \in \{1, \ldots, k\} \\ E = H_k(x_1, \ldots, x_k, n_1, \ldots, n_k) \end{cases}$$

or

$$\begin{cases} \dot{n}_i = r(x_i, E)n_i \quad \forall i \in \{1, \ldots, k\} \\ \dot{E} = H_k(x_1, \ldots, x_k, n_1, \ldots, n_k, E) \end{cases}$$

(c) Give the invasion fitness $s_E(y)$ and determine the essential dimension of the environment. What is the maximum number of different resident strategies that can generically coexist in a log-bounded manner.

(d) Give an expression of the invasion fitness (or fitness proxy) explicitly in terms of the resident strategies if the resident environment is saturated (i.e., give $s_{\mathbf{x}}(y)$, or $\tilde{s}_{\mathbf{x}}(y)$ for its proxy, where $\mathbf{x}$ is a vector of coexisting resident strategies).

(e) Is there an optimisation principle, i.e., is there a function of the environment that is minimised or maximised by the strategy dynamics? What is it?

Exercise 1:

$$\begin{cases} \dot{S} = \lambda(N)S - \mu S - S\sum_j \beta(x_j)I_j \\ \dot{I}_i = S\beta(x_i)I_i - \mu I_i - \alpha(x_i)I_i \end{cases}$$

for $i \in \{1, \ldots, k\}$ and with $N = S + \sum_j I_j$.

Exercise 2:

$$\begin{cases} \dot{S} = \lambda N - \mu(N)S - S\sum_j \beta(x_j)I_j \\ \dot{I}_i = S\beta(x_i)I_i - \mu(N)I_i - \alpha(x_i)I_i \end{cases}$$

for $i \in \{1, \ldots, k\}$ and with $N = S + \sum_j I_j$.

Exercise 3:

$$\begin{cases} \dot{N}_i = \alpha(x_i)N_i\left(1 - \frac{\sum_j N_j}{K}\right) - \frac{\beta(x_i)N_i}{1+\sum_j T(x_j)\beta(x_j)N_j}P \\ \\ \dot{P} = P\left(\sum_i \frac{\beta(x_i)N_i}{1+\sum_j T(x_j)\beta(x_j)N_j}\right) - \delta P \end{cases}$$

for $i \in \{1, \dots, k\}$. Does it matter for the strategy dynamics what the dynamics of P precisely looks like? Why not, or why and how?

Exercise 4:

$$\dot{n}_i = \alpha(x_i)n_i - \delta(x_i)n_i - \gamma \sum_j \left(\frac{x_i}{x_i + x_j} n_i n_j\right)$$

for $i \in \{1, \dots, k\}$.