Topology II — Homework set 4

Spring 2015 — Tue 10.2.2015

Exercise 1

Prove the following Theorem ("Lause 4.8" in the textbook):

Let X have the topology induced by the function $f : X \to Y$ from Y. If $\mathcal{B}$ is a base for the topology of Y, the sets $f^{\leftarrow}B$, for $B \in \mathcal{B}$, form a base for X. If $\mathcal{A}$ is a subbase for the topology of Y, the sets $f^{\leftarrow}A$, for $A \in \mathcal{A}$, form a subbase for X.

Exercise 2

Consider a topological space $(X, \mathcal{T})$ and $A \subset X$. As in the textbook, call A a *discrete set* in X if every point in A is an isolated point (*erakkopiste*) in the relative topology inherited from X. Show that A is a discrete set in X if and only if its relative topology $\mathcal{T}|_A$ is discrete (meaning that $\mathcal{T}|_A = \mathcal{P}(A)$).

Exercise 3

Suppose $(X, \mathcal{T})$ is a topological space and $A \subset B \subset X$. Show that, if A is dense in B (in the relative topology $\mathcal{T}|_B$), then A is dense in the closure of B, in $\mathrm{cl}_X B$.

Exercise 4

Define a function $\mathrm{pr}_1 : \mathbb{R}^2 \to \mathbb{R}$ by the formula $\mathrm{pr}_1(\mathbf{x}) := x_1$ (meaning that pr_1 is a projection, as defined in Topology I). Then pr_1 induces a topology on $\mathbb{R}^2$ from the ordinary topology of $\mathbb{R}$. Determine the following sets using this topology: $\mathrm{cl}\, B$, $\mathrm{int}\, B$, $\mathrm{ext}\, B$ and ∂B, where $B := B(\mathbf{0}, 1) \subset \mathbb{R}^2$ is the unit ball in the plane.

(*Hint:* You can find the closures of B and $\complement B$ by applying Theorem 4.5 of the textbook.)

Exercise 5

Show that an injective function $f : X \to Y$ is an embedding if and only if f induces the topology of X from Y.