Logic I
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Exercises 9

Read chapters 2.10–2.11 on substitution and the deduction rules for the universal quantifier.

1. (Recap) Let $\mathcal{M} = (M, R^{\mathcal{M}})$, where

- $M = \{2, 3, 4, 5, 6\}$ and
- $(a, b) \in R^{\mathcal{M}}$ iff a divides b.

Which set does the formula

(1) $\exists x(R(x, y) \wedge \neg x = y)$
(2) $\exists y(R(x, y) \wedge \neg x = y)$

define in the model?

2. Is the term t free for the variable x in the formula A when

(1) t is y and A is $\exists y R(x, y)$
(2) t is y and A is $\exists x R(x, z)$
(3) t is y and A is $\exists z R(x, y)$
(4) t is z and A is $\exists z P(z) \wedge R(x, y)$
(5) t is z and A is $\exists z P(z) \wedge R(x, z)$

If the substitution is allowed, what is $A(t/x)$?

3. Prove the following special case of the Substitution Lemma: Let A be the formula $\forall z(R_0(y, z) \to P_0(z))$. Let t be the variable x. Then the following are equivalent for all models $\mathcal{M}$ and assignments s:

(1) $\mathcal{M} \models_s A(t/y)$
(2) $\mathcal{M} \models_{s(a/y)} A$, where $a = t^{\mathcal{M}}\langle s \rangle$.

4. Derive by natural deduction the sentence $\forall x R_0(x, x)$ from the sentence $\forall x \forall y R_0(x, y)$.

5. Derive the sentence $\neg \forall x P_0(x)$ from the sentence $\forall x \neg P_0(x)$.

6. Derive the sentence $\forall y P_1(y)$ from the sentences $\forall x P_0(x)$ and $\forall x(\neg P_1(x) \to \neg P_0(x))$.