Logic I
Department of Mathematics and Statistics, University of Helsinki
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Exercises 8

Read chapters 2.8–2.9 on free and bound variables and definability.

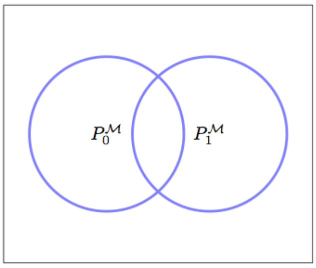
1. (Recap) Show that the formula $\neg\forall x P(x) \to \exists x \neg P(x)$ is valid.

2. Let A be a formula and $\mathcal{M}$ a model. Show that if s and s' are assignments that agree on every variable occurring free in A, then s satisfies A in $\mathcal{M}$ if and only if s' does. (Hint: Use induction on A.)

3. Which occurrences of variables are free and which are bound in the following formulas? Which formulas are sentences?

(a) $\forall x(P_0(x) \to P_1(y))$
(b) $\forall x(\forall y\, xEy \vee \forall z\, yEz)$
(c) $\forall y(\exists x\, x < y \vee \exists x\, y < x)$
(d) $\forall x(P(x) \to \exists y R(x,y))$

4. Indicate the set defined by the formula $P_0(x) \wedge \neg P_1(x)$ in the unary structure below.



5. Draw the binary relation defined by the formula

$$d < x \ \vee \ y < c$$

in the structure $\mathcal{M} = (\mathbb{R}, <, 0, 1)$, $c^{\mathcal{M}} = 0$, $d^{\mathcal{M}} = 1$.

6. An element a is definable in a model $\mathcal{M}$ if the set $\{a\}$ is definable in $\mathcal{M}$. Show that 2 is definable in the model $(\mathbb{N}, <)$, where $<$ is the natural order on the natural numbers.