Logic I
Department of Mathematics and Statistics, University of Helsinki
Spring 2015
Exercises 7

Read chapters 2.5–2.7 on predicate logic formulas and validity.

1. Which of the assignments below satisfy the formula $P_0(x) \leftrightarrow R(x, y)$ in the model in figure 1? Justify your answer.

	x	y	z
s_0	1	2	6
s_1	3	4	6
s_2	5	6	7

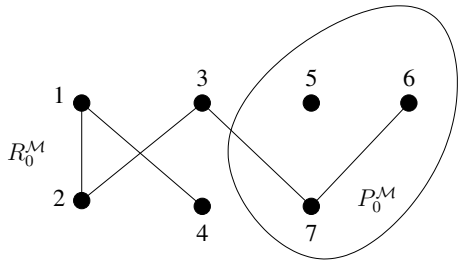

FIGURE 1. A model

2. Which of the following assignments satisfy the formula xEy in the graph in figure 2? Which satisfy the formula $\exists y\ xEy$? Justify.

	x	y	z
s_0	1	2	3
s_1	4	5	6
s_2	3	8	7

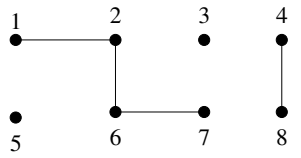

FIGURE 2. A graph

3. Give a $\{E\}$-formula that expresses that each vertex in a graph has at least two different neighbours. Two vertices in a graph are neighbours if there is an edge between them.

4. Show that the formulas $\neg\forall xA$ and $\exists x\neg A$ are logically equivalent.

5. Show that the formula $\forall y\exists xA$ is a logical consequence of the formula $\exists x\forall yA$.

6. Show that the formulas $\exists x\forall yR(x, y)$ and $\forall y\exists xR(x, y)$ are not logically equivalent.