

Logic I

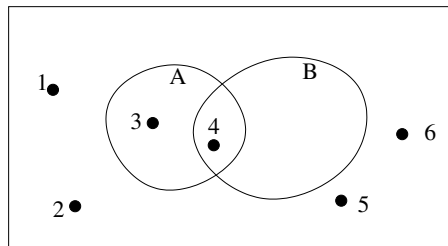
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Exercises 6

Read chapters 2.2–2.4 on structures and atomic formulas of predicate logic.

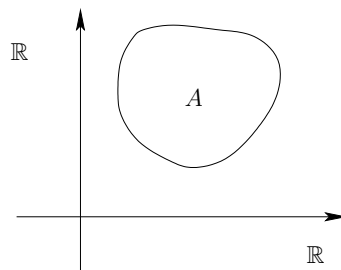
1. Let \mathcal{M} be the following structure:



- For which vocabulary is \mathcal{M} a model?
- What is $\text{dom}(\mathcal{M})$?
- Which are the interpretations of the symbols of the vocabulary?

Are the answers unique?

2. Let \mathcal{M}' be the following structure:



- For which vocabulary is \mathcal{M}' a model?
- What is $\text{dom}(\mathcal{M}')$?
- Which are the interpretations of the symbols of the vocabulary?

Are the answers unique?

3. Give an example of a model for the vocabulary L where

- $L = \{P_0, P_1\}$,
- $L = \{R_0, R_1, c_0\}$,
- $L = \{P_0, P_1, R_0, c_0, c_1, c_2\}$.

4. Let $L = \{P_0, P_1\}$ and $L' = \{P_0, c_0, c_1\}$. Give examples of models \mathcal{M} and \mathcal{M}' , such that all the following hold:

- \mathcal{M} is a model for L and \mathcal{M}' is a model for L' ,
- $\text{dom}(\mathcal{M}) = \text{dom}(\mathcal{M}')$
- $P_0^{\mathcal{M}'} = P_1^{\mathcal{M}} \neq P_0^{\mathcal{M}}$
- no assignment satisfies the formula $P_0(c_1)$ in the model \mathcal{M}' .

5. Which of the following assignments satisfy the formula $P_0(x)$ in the structure in figure 1? Justify your answer.

	x	y	z
s_0	a	a	b
s_1	c	d	e
s_2	b	c	g

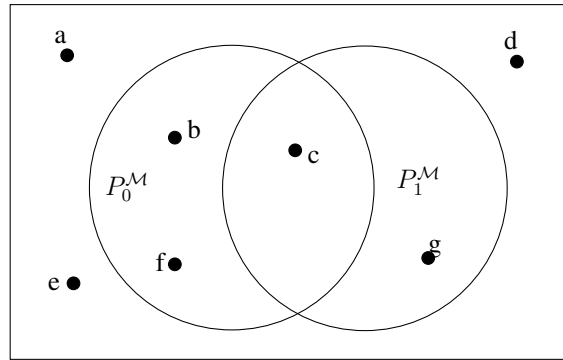


FIGURE 1. A model

6. Which of the following assignments satisfy the formula xEy in the graph in figure 2? Justify your answer.

	x	y	z
s_0	a	c	e
s_1	b	d	f
s_2	e	d	c

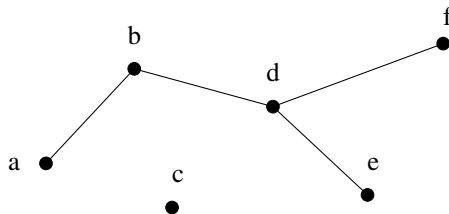


FIGURE 2. A graph