Logic I
Department of Mathematics and Statistics, University of Helsinki
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Exercises 5

Read chapters 1.10–1.11 on the soundness of natural deduction and on semantic trees.

1. (a) How would you prove that a given deduction exists?
(b) How would you prove that a given deduction does not exist?
(c) Prove that $\{(p_0 \wedge p_1) \to p_2\} \nvdash (p_0 \to p_2) \wedge (p_1 \to p_2)$.
(d) Prove that $\{(p_0 \to p_2) \vee (p_1 \to p_2)\} \vdash (p_0 \wedge p_1) \to p_2$.

2. Is it possible to deduce the formula $((p_0 \wedge p_1) \to \neg p_0) \to (\neg p_0 \vee p_1)$ with natural deduction? Give a proof for your answer.

3. Is it possible to deduce the formula $\neg p_0 \vee p_1$ from the formula $p_0 \to (p_1 \vee \neg p_0)$ with natural deduction? Give a proof for your answer.

4. Is it possible to deduce the formula $p_2 \to \neg(p_0 \wedge p_1)$ from the formula $(p_0 \to \neg p_2) \vee (\neg p_1 \to \neg p_2)$ with natural deduction? Give a proof for your answer.

5. Give a semantic proof for the formulas

(a) $(A \vee (B \to C)) \to (B \to (\neg A \to C))$ and
(b) $(A \wedge (B \vee C)) \to ((A \wedge B) \vee C)$.

6. Use a semantic tree to find a valuation v for which $v(((p_0 \wedge p_1) \to p_2) \to ((p_0 \to p_2) \wedge (p_1 \to p_2))) = 1$. Why does this not contradict 1(c) above?