Logic I
Department of Mathematics and Statistics, University of Helsinki
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Exercises 4

Read chapters 1.6–1.9 on natural deduction in the course material.

1. (a) Derive by natural deduction $A \wedge (B \vee C)$ from $A \wedge C$.
(b) Derive by natural deduction $A \to D$ from $A \to B$, $B \to C$ and $C \to D$.

2. Which of the following are correct natural deductions? In the correct cases, which are the assumptions and the conclusion and which rules were applied? In the incorrect cases, find the errors.

$$\dfrac{\dfrac{\dfrac{[A \to (B \wedge C)] \qquad A}{B \wedge C}}{C}}{A \to C}$$

(a)

$$\dfrac{\dfrac{[p_0 \wedge p_1]}{p_1}}{(p_0 \wedge p_1) \to p_1}$$

(b)

$$\dfrac{\begin{matrix}[A] & \quad & [B] \\ \vdots & & \vdots \\ C & & C\end{matrix}}{(A \vee B) \to C}$$

(c)

$$\dfrac{\dfrac{A \wedge B}{B}}{A \to B}$$

(d)

3. Derive by natural deduction the formula $A \vee B$ from the formula $A \vee (B \vee A)$.

4. Derive by natural deduction the formula $\neg(\neg B \wedge \neg C)$ from the formula $A \wedge (B \vee C)$.

5. (a) Construct a natural deduction of the formula $A \vee (A \to B)$ formalizing the following reasoning:

> Assume towards a contradiction that the claim is false. Then if A is true, also $A \vee (A \to B)$ is, which contradicts our counter assumption, so A is false. With little effort we see that then $A \to B$ is true, so also $A \vee (A \to B)$ is true, which again contradicts our assumption. So the assumption was false and the claim has to be true.

(b) Can you think of a way to shorten the deduction?

6. Derive the formula $(A \to B) \vee (B \to A)$ using natural deduction.