Logic I
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Exercises 3

Read chapter 1.5 on truth functions in the course material. You can look up conjunctive normal form on the internet.

1. Give a valuation that proves the following claim or prove that no such valuation exists:

(a) The propositional formula $(\neg p_0 \vee p_1) \leftrightarrow (p_0 \wedge p_1)$ is satisfiable.
(b) The propositional formula $(p_0 \to p_1) \wedge \neg(\neg p_0 \vee p_1)$ is satisfiable.
(c) The propositional formula $((p_0 \to p_1) \to p_1) \to p_1$ is falsifiable.
(d) The propositional formula $(p_0 \leftrightarrow p_1) \leftrightarrow (p_0 \leftrightarrow (p_1 \leftrightarrow p_0))$ is falsifiable.

2. Come up with two propositional formulas both of which contain at least two propositional symbols and such that they are not logically equivalent but one is the logical consequence of the other.

3. (a) Let A be the propositional formula $p_0 \wedge \neg p_0$. What is the truth function f_A?
(b) What can you say about the truth functions of tautologies, contradictions and contingent formulas?

4. Are the following formulas in

(i) disjunctive normal form?
(ii) conjunctive normal form?

(a) p_0
(b) $p_0 \wedge \neg p_0$
(c) $p_0 \vee \neg p_0$
(d) $(p_0 \wedge p_1) \vee p_0$
(e) $(\neg p_0 \vee p_1) \wedge (\neg p_0 \vee \neg p_1)$
(f) $p_0 \wedge p_1 \wedge p_2$
(g) $p_0 \vee p_1 \vee p_2$
(h) $(p_0 \wedge p_1) \vee \neg(p_2 \wedge p_1)$

5. Give a propositional formula A in disjunctive normal form such that the truth function f_A it defines is:

x_0	x_1	x_2	$f(x_0, x_1, x_2)$
1	1	1	1
1	1	0	0
1	0	1	0
1	0	0	0
0	1	1	1
0	1	0	0
0	0	1	1
0	0	0	0

6. Prove that $\{\rightarrow\}$ is not a universal set of connectives.

In the following exercise we have a look at natural deduction. To solve the problem a preliminary look at chapter 1.6 should suffice.

7. We examine the "deduction" below:

$$\dfrac{\dfrac{A \vee B}{A} \qquad \dfrac{A \vee B}{B}}{A \wedge B}$$

(a) Find the errors of the deduction.
(b) Come up with two English sentences A and B and using them explain what happens in the "deduction".