Logic I
Department of Mathematics and Statistics, University of Helsinki
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Exercises 2

Read chapters 1.3–1.4 on valuations and truth tables in the course material.

1. Prove that each propositional formula with an even number of symbols must contain a negation. (Note that a propositional symbol is counted as one symbol, so e.g. p_{315} is one symbol.)

2. Assume that $v(p_0) = 1$, $v(p_1) = 0$ and $v(p_2) = 0$. Compute

(a) $v((p_0 \vee p_1) \leftrightarrow (p_2 \wedge \neg p_0))$,
(b) $v(p_0 \rightarrow (p_1 \rightarrow p_2))$,
(c) $v(\neg(\neg p_0 \rightarrow \neg p_1) \vee \neg(\neg p_1 \rightarrow \neg p_2))$.

3. Give an example of a valuation, for which the formula:

(a) $(p_0 \wedge (\neg p_1 \wedge (\neg p_2 \wedge (p_3 \wedge \neg p_4))))$
(b) $\neg((p_0 \wedge p_1) \rightarrow (\neg p_0 \wedge \neg p_1))$
(c) $(\neg p_0 \wedge ((p_1 \rightarrow p_2) \vee (p_3 \leftrightarrow \neg p_4)))$

is true.

4. In the lecture we considered the following example:

> If it rains and is windy outdoors then the ones sitting by the window get wet, unless the window is closed.

In the lecture we denoted the atomic formulas:

> p_0: It rains outdoors. p_1: It is windy outdoors. p_2: The ones sitting by the window get wet. p_3: The window is closed.

(1) Draw a truth table where for each valuation you determine the truth value of the sentence based on your intuitive interpretation of the English language.
(2) In the lecture the following formalizations were suggested for the sentence:
- $((p_0 \wedge p_1) \wedge \neg p_3) \rightarrow p_2$
- $(p_0 \wedge p_1) \rightarrow (p_2 \vee (\neg p_2 \wedge \neg p_3))$
- $\neg p_3 \rightarrow ((p_0 \wedge p_1) \rightarrow p_2)$

Draw the truth tables for the formulas. Which ones are equivalent? Does some of them correspond to your intuitive interpretation of the sentence? If not, how would you formalize the sentence.

5. Use the truth table method to decide whether

(a) $p_0 \to \neg p_0$
(b) $p_0 \vee \neg(p_0 \wedge p_1)$

is a tautology, contingency or contradiction.

6. Use the truth table method to prove the equivalence of

(a) $A \to B$ and $\neg A \vee B$
(b) $A \leftrightarrow B$ and $(A \to B) \wedge (B \to A)$