Logic I
Department of Mathematics and Statistics, University of Helsinki
Spring 2015
Exercises 1

These exercises are solved together in the exercises class on 22.1. and treated again the following week.

Read chapter 1.2 in Jouko Väänänen's material Logic One (propositional formulas).

1. Denote by the propositional symbols p_0, p_1 and p_2 the following sentences

> p_0: It is raining.
> p_1: It is windy.
> p_2: It is cold.

Write the following sentences as propositional formulas:

(a) If it is raining and windy then it is cold.
(b) If it is cold but it isn't raining, then it is windy.
(c) If it isn't raining, then it isn't cold, except if it is windy.

2. Consider the sentence "Either Harry buys oranges and bananas or Therese buys tomatoes, but neither buys cucumber."

(a) Which are the atomic sentences?
(b) Write the sentence as a propositional formula.

3. Come up with an example sentence in natural language which written as a propositional formula has the form

$$((p_0 \to (p_1 \lor p_2)) \land \neg p_2) \to (p_0 \to p_1).$$

4. Which of the following strings of symbols are propositional formulas? How would you motivate your answer?

(a) $((\neg p_0 \land p_1) \to p_0)$
(b) $p_0 \to p_1 \to p_2$
(c) $))p_0 \lor \to p_2$
(d) p_{3201}
(e) $p_0 \lor p_1 \land p_2$
(f) $((p_0 \land p_0) \land \neg p_0)$

5. Which are the main connective and immediate subformulas of the following propositional formulas?

(a) $\neg(p_0 \wedge p_1)$
(b) $((\neg(p_0 \rightarrow p_1) \rightarrow p_1) \wedge (p_0 \vee \neg p_1))$
(c) $((p_0 \rightarrow (p_1 \rightarrow p_2)) \wedge \neg p_1)$

6. Which are the subformulas of the propositional formula $((p_0 \vee (\neg p_1 \wedge p_2)) \leftrightarrow (\neg p_1 \rightarrow (p_2 \wedge p_3)))$?

7. Give an example of strings of symbols A and B such that these *are not* propositional formulas but the formula AB is (AB is the string you get by writing A and B one after the other).

8. Give an example of strings A and B such that these are not propositional formulas but the string $(A \vee B)$ is.