Logic I
Department of Mathematics and Statistics, University of Helsinki
Spring 2015
Exercises 12

Read chapters 2.19–2.22 on n-ary relations, functions and isomorphisms.

1. Deduce the sentence

$$\forall x\exists y(R_0(F_0^1(x),x)\vee R_1(y,x))$$

from the sentence

$$\forall x\forall y(R_0(y,x)\vee R_1(y,x)).$$

2. Consider the sentence:

$$(\forall xR_0^3(x,c,d)\vee\forall yP_0(y))\to\forall x(R_0^3(x,c,d)\vee P_0(F_0^1(x))).$$

Is the sentence valid, contingent of a contradiction? If it is valid or a contradiction, demonstrate this with a suitable deduction or semantic tree. If it is contingent, demonstrate this with models obtained by means of semantic trees.

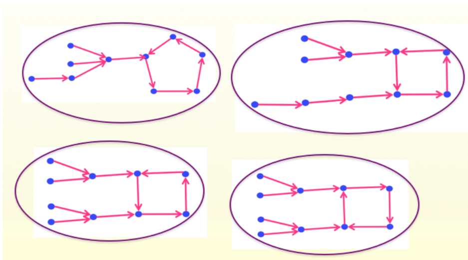
3. There is an error in the following "deduction" of the sentence $\forall y\exists z\forall xR_0^3(z,y,x)$ from the sentence $\forall y\forall xR_0^3(F_0^1(x),y,x)$:

$$\cfrac{\cfrac{\cfrac{\forall y\forall xR_0^3(F_0^1(x),y,x)}{\forall xR_0^3(F_0^1(x),y,x)}\ \forall\,\mathbf{E}}{\exists z\forall xR_0^3(z,y,x)}\ \exists\,\mathbf{I}}{\forall y\exists z\forall xR_0^3(z,y,x)}\ \forall\,\mathbf{I}$$

What is the error?

4. Is it possible to deduce the sentence $\exists x_0\exists x_1R_0(x_0,x_1)$ from the sentences $\forall x_0(P_0(F_0^1(x_0))\to R_0(x_0,F_0^1(x_0)))$ and $\exists x_1P_0(x_1)$ with natural deduction?

5. Which of the following models each with a unary function are isomorphic?



6. Let $L = \{P_0, c_0\}$, where P_0 is a unary predicate symbol and c_0 is a constant symbol. Further let

$$\mathcal{M}_1 = (\mathbb{Z}, P_0^{\mathcal{M}_1}, c_0^{\mathcal{M}_1}),$$

where

$$P_0^{\mathcal{M}_1} = \mathbb{N} \quad \text{and} \quad c_0^{\mathcal{M}_1} = 1$$

and

$$\mathcal{M}_2 = (\mathbb{N}, P_0^{\mathcal{M}_2}, c_0^{\mathcal{M}_2}),$$

where

$$P_0^{\mathcal{M}_2} = \{2k : k \in \mathbb{N}\} \quad \text{and} \quad c_0^{\mathcal{M}_2} = 1.$$

Are $\mathcal{M}_1$ and $\mathcal{M}_2$ isomorphic?

7. Let $L = \{P, R\}$ be a vocabulary. Let M be the set $\{0, 1, \ldots, 9\}$ (the natural numbers from zero to nine). We define two L-structures $\mathcal{M}_1$ and $\mathcal{M}_2$ as follows: Both have the set M as universe,

$$P^{\mathcal{M}_1} = \{0, 1, 2, 3, 4\} \quad \text{and} \quad P^{\mathcal{M}_2} = \{5, 6, 7, 8, 9\},$$

and

$$R^{\mathcal{M}_1} = R^{\mathcal{M}_2} = \{(2, 6), (6, 2), (2, 8)(8, 2), (6, 8), (8, 6)\}.$$

Are the structures $\mathcal{M}_1 = (M, P^{\mathcal{M}_1}, R^{\mathcal{M}_1})$ and $\mathcal{M}_2 = (M, P^{\mathcal{M}_2}, R^{\mathcal{M}_2})$ isomorphic?

8. Let $L = \{P, R\}$. Let $\mathcal{M}$ and $\mathcal{M}'$ be two L-structures, with domain $\mathrm{dom}(\mathcal{M}) = \mathrm{dom}(\mathcal{M}') = \{1, 2, 3, 4, 5, 6, 7\}$. Further let

$$P^{\mathcal{M}} = \{1, 2, 3, 4\} \quad \text{and} \quad R^{\mathcal{M}} = \{(2, 5), (5, 7), (7, 2)\}$$

and

$$P^{\mathcal{M}'} = \{4, 5, 6, 7\} \quad \text{and} \quad R^{\mathcal{M}'} = \{(2, 3), (3, 4), (4, 2)\}.$$

Are $\mathcal{M}$ and $\mathcal{M}'$ isomorphic?