Logic I
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Exercises 11

Read chapters 2.15–2.16 on axioms and semantic trees.

1. Deduce

$$\forall x(P(x) \to \exists y(x = y \land P(y))).$$

2. Deduce from the axioms of order

$$\forall x_0 \forall x_1 \forall x_2 (x_0 < x_1 \to (x_2 < x_1 \lor x_0 < x_2))$$

3. Deduce from the axioms of graphs

$$\forall x \forall y (xEy \to \neg x = y).$$

4. Can one deduce $\neg \forall x \neg xEx$ from the graph axioms? And how about $\forall x \exists y \neg xEy$?

5. Use the method of semantic trees to find a model for

$$\forall x \exists y \forall z (R_0(x, y) \land R_0(y, z) \land \neg \forall x \forall y R_0(x, y)).$$

Note that there are also other methods to find models, like guessing and testing.

6. Give a semantic proof for

$$\exists x \forall y (R_0(x, y) \lor P_0(x)) \to \forall y \exists x (P_0(x) \lor R_0(x, y))$$

7. Give a semantic proof for

$$\exists x \forall y \neg R_0(x, y) \to \exists x \neg \exists y R_0(x, y)$$

8. Consider the formula:

$$(R_0(x, y) \lor \forall x P(x)) \to \forall x (R_0(x, y) \lor P(x)).$$

Is the formula valid, contingent of a contradiction? If it is valid or a contradiction, demonstrate this with a suitable deduction or semantic tree. If it is contingent, demonstrate this with models obtained by means of semantic trees.