Logic I
Department of Mathematics and Statistics, University of Helsinki
Spring 2015
Exercises 10

Read chapters 2.11–2.15 on natural deduction, soundness and axioms.

1. Derive

$$\forall x \exists y R_0(x, y) \wedge \forall x \exists y R_1(x, y)$$

from

$$\forall x \exists y (R_0(x, y) \wedge R_1(x, y)).$$

2. Derive

$$\neg(\forall x P_0(x) \wedge \exists x \neg P_0(x)).$$

3. Derive $\exists x P_0(x) \to \exists y P_1(y)$ from the formula $\exists y \forall x (P_0(x) \to P_1(y))$.

4. Derive $\neg \exists y P_0(y) \to \neg P_0(c)$.

5. Show that the following sentence cannot be derived by natural deduction:

$$\exists x \neg P_0(x) \to \neg \exists x P_0(x)$$

6. Show that the following sentence cannot be derived by natural deduction:

$$\forall z (\forall x R_0(x, x) \to \forall y R_0(z, y))$$

7. Show that the following sentence cannot be derived by natural deduction:

$$\forall x (P_0(x) \to \forall y P_0(y))$$

8. Derive $\exists x P(x) \to \forall x P(x)$ from the sentence $\forall x \forall y\, x = y$. Hint: use identity axioms.