Linear Algebra Part II
Second exercise
For following exercises determine whether S is a subspace of given $\mathbb{R}^n$.

1. $S = \{(x, y)|x = 0\} \subseteq \mathbb{R}^2$
2. $S = \{(x, y)|y = 2x\} \subseteq \mathbb{R}^2$
3. $S = \{(x, y)|x \geq 0, y \geq 0\} \subseteq \mathbb{R}^2$
4. $S = \{(x, y)|xy \geq 0\} \subseteq \mathbb{R}^2$
5. $S = \{(x, y, z)|x = y = z\} \subseteq \mathbb{R}^3$
6. $S = \{(x, y, z)|x = y = z\} \subseteq \mathbb{R}^3$
7. $S = \{(x, y, z)|z = 2x, y = 0\} \subseteq \mathbb{R}^3$
8. $S = \{(x, y, z)|x - y + z = 1\} \subseteq \mathbb{R}^3$
9. $S = \{(x, y, z)||x - y| = |y - z|\} \subseteq \mathbb{R}^3$
10. Prove that the only nontrivial subspaces of $\mathbb{R}^2$ are lines that contain origin.
11. Prove that the only nontrivial subspaces of $\mathbb{R}^3$ are lines and planes that contain origin.
12. Prove that intersection of any two subspaces is a subspace.
13. With two examples show that union of two subspaces can be subspace of not. Can you find necessary and sufficient condition such that union of two subspaces be a subspace?

    In next two exercise determine whether $\mathbf{b}$ is in $col(A)$, whether $\mathbf{w}$ is in $row(A)$ and whether $\mathbf{v}$ is in $null(A)$.

14. $A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 1 & 1 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$, $\mathbf{w} = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} -1 \\ 3 \\ -1 \end{bmatrix}$
15. $\begin{bmatrix} 1 & 1 & -3 \\ 0 & 2 & 1 \\ 1 & -1 & -4 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$, $\mathbf{w} = \begin{bmatrix} 2 & 4 & -5 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} 7 \\ -1 \\ 2 \end{bmatrix}$

    In next two exercises find a basis for $row(A)$, $col(A)$ and $null(A)$

16. $A = \begin{bmatrix} 1 & 1 & -3 \\ 0 & 2 & 1 \\ 1 & -1 & -4 \end{bmatrix}$
17. $A = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & -1 & 1 \\ 0 & 1 & -1 & -1 \end{bmatrix}$

    In next three exercise determine whether the given vectors form a basis for $\mathbb{R}^3$. If not find a basis for the span of the vectors.

18. $\begin{bmatrix} 1 & -1 & 0 \end{bmatrix}$, $\begin{bmatrix} -1 & 0 & 1 \end{bmatrix}$, $\begin{bmatrix} 0 & 1 & -1 \end{bmatrix}$
19. $\begin{bmatrix} 1 & 1 & 0 \end{bmatrix}$, $\begin{bmatrix} 1 & 0 & 1 \end{bmatrix}$, $\begin{bmatrix} 0 & 1 & 1 \end{bmatrix}$
20. $\begin{bmatrix} 1 & -1 & 3 \end{bmatrix}$, $\begin{bmatrix} -1 & 5 & 1 \end{bmatrix}$, $\begin{bmatrix} 1 & -3 & 1 \end{bmatrix}$

On Monday 23 March we did go through the rest of Sec 3.5 (Subspaces, Basis, etc). Just a couple of examples and second version of Fundamental theorem of invertible matrices left that we will visit them on Friday. If we have time we are going to start next section (Linear Transformations).