Linear Algebra
Second exercise:
In exercise 1, 2 and 3 find $\mathbf{u}.\mathbf{v}$ and $\|\mathbf{u}\|$. Also give a unit vector in the direction of $\mathbf{u}$.

1. $\mathbf{u} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$ 2. $\mathbf{u} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$

3. $\mathbf{u} = \left[1, \sqrt{2}, \sqrt{3}, 0\right]$, $\mathbf{v} = \left[4, -\sqrt{2}, 0, -5\right]$

4. If $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$ are vectors in $\mathbb{R}^n$, $n \geq 2$ and c is a scalar, explain why the following expressions make no sense:

(a) $\|\mathbf{u}.\mathbf{v}\|$ (b) $\mathbf{u}.\mathbf{v} + \mathbf{w}$
(c) $\mathbf{u}.(\mathbf{v}.\mathbf{w})$ (d) $c.(\mathbf{u} + \mathbf{v})$

Find the angle between $\mathbf{u}$ and $\mathbf{v}$ in next three exercise.

5. $\mathbf{u} = \begin{bmatrix} 3 \\ 0 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$ 6. $\mathbf{u} = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} 1 \\ -2 \\ -1 \end{bmatrix}$

7. $\mathbf{u} = [4, 3, -1]$, $\mathbf{v} = [1, -1, 1]$

8. Let $A = (1, 1, -1)$, $B = (-3, 2, -2)$, $C = (2, 2, -4)$. Prove that $\triangle ABC$ is a right-angled tirangle.

In exercise 9 - 12, find the projection of $\mathbf{v}$ onto $\mathbf{u}$. Draw a sketch in exercise 9 and 10.

9. $\mathbf{u} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$ 10. $\mathbf{u} = \begin{bmatrix} \frac{3}{5} \\ -\frac{4}{5} \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

11. $\mathbf{u} = \left[\frac{2}{3}, -\frac{2}{3}, -\frac{1}{3}\right]$, $\mathbf{v} = [2, -2, 2]$

12. $\mathbf{u} = [1, -1, 1, -1]$, $\mathbf{v} = [2, -3, -1, -2]$

13. Show that there are no vectors $\mathbf{u}$ and $\mathbf{v}$ such that $\|\mathbf{u}\| = 1$, $\|\mathbf{v}\| = 2$ and $\mathbf{u}.\mathbf{v} = 3$.

14. (a) Prove that $\mathrm{proj}_{\mathbf{u}}(\mathrm{proj}_{\mathbf{u}}(\mathbf{v})) = \mathrm{proj}_{\mathbf{u}}(\mathbf{v})$.
(b) Prove that $\mathrm{proj}_{\mathbf{u}}(\mathbf{v} - \mathrm{proj}_{\mathbf{u}}(\mathbf{v})) = 0$.
(c) Explain (a) and (b) geometrically.