Linear Algebra

Fourth exercise:

In the following exercises write the matrices of coefficent and augmented of given system of linear equations and solve them with Gaussian and Gauss-Jordan elimination.

1. $x - y = 0$
$2x + y = 3$

2. $2x_1 + 3x_2 - x_3 = 1$
$x_1 + x_3 = 0$
$-x_1 + 2x_2 - 2x_3 = 0$

3. $x + 5y = -1$
$-x + y = -5$
$2x + 4y = 4$

4. $a - 2b + d = 2$
$-a + b - c - 3d = 1$

5. (a) Find a system of two linear equations in the variables x and y whose solution set is given by the parametric equations $x = t$ and $y = 3 - 2t$.

(b) Find another parametric solution to the system in part (a) in which the parameter is s and $y = s$.

(c) Is the solution to part (a) unique? If yes, why? If not find another solution.

6. (a) Find a system of two linear equations in the variables x_1, x_2 and x_3 whose solution set is given by the parametric equations $x_1 = t$, $x_2 = 1 + t$ and $x_3 = 2 - t$.

(b) Find another parametric solution to the system in part (a) in which the parameter is s and $x_3 = s$.

(c) Is the solution to part (a) unique? If yes, why? If not find another solution.

In next exercises use elementary row operations to reduce the given matrix to (a) row echelon form and (b) reduced row echelon form. Find the rank of each matrix.

7. $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

8. $\begin{bmatrix} 3 & 5 \\ 5 & -2 \\ 2 & 4 \end{bmatrix}$

9. $\begin{bmatrix} 2 & -4 & -2 & 6 \\ 3 & 1 & 6 & 6 \end{bmatrix}$

10. $\begin{bmatrix} 3 & -2 & -1 \\ 2 & -1 & -1 \\ 4 & -3 & -1 \end{bmatrix}$

11. $\begin{bmatrix} -2 & -4 & -7 \\ -3 & -6 & 10 \\ 1 & 2 & -3 \end{bmatrix}$

12. $\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & y_1 \\ 1 & 1 & 1 & 0 & 0 & y_2 \\ 0 & 1 & 1 & 1 & 0 & y_3 \\ 0 & 0 & 1 & 1 & 1 & y_4 \\ 0 & 0 & 0 & 1 & 1 & y_5 \end{bmatrix}$

13. (a) Give an example of three planes that have a common line of intersection.

(b) Give an example of three planes that inrersect in pairs but have no common point of intersection.

(c) Give an example of three planes, exactly two of which are parallel.

(d) Give an example of three planes that inrersect in a sinle point.

In next exercises solve the system of linear equations over the indicated $\mathbb{Z}_p$.

14. $x + 2y = 1$
$x + y = 2$ over $\mathbb{Z}_3$

15. $x + y = 1$
$y + z = 0$
$x + z = 1$ over $\mathbb{Z}_2$

16. $x + y = 1$
$y + z = 0$
$x + z = 1$ over $\mathbb{Z}_3$

17. $3x + 2y = 1$
$y + 4y = 1$ over $\mathbb{Z}_5$