Linear Algebra

Fifth exercise:

In the following exercises determine if the vector $\mathbf{v}$ is a linear combination of the remaining vectors.

1. $\mathbf{v} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, $\mathbf{u_1} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$, $\mathbf{u_2} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$

2. $\mathbf{v} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$, $\mathbf{u_1} = \begin{bmatrix} 4 \\ -2 \end{bmatrix}$, $\mathbf{u_2} = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$

3. $\mathbf{v} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, $\mathbf{u_1} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$, $\mathbf{u_2} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$

4. $\mathbf{v} = \begin{bmatrix} 3 \\ 2 \\ -1 \end{bmatrix}$, $\mathbf{u_1} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$, $\mathbf{u_2} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$

In next two exercises determine if the vector $\mathbf{b}$ is in the span of the columns of the matrix $\mathbf{A}$.

5. $\mathbf{A} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} 5 \\ 6 \end{bmatrix}$

6. $\mathbf{A} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} 10 \\ 11 \\ 12 \end{bmatrix}$

7. Show that $\mathbb{R}^2 = Span\left(\begin{bmatrix} 3 \\ -2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}\right)$

8. Show that $\mathbb{R}^3 = Span\left(\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}\right)$

In next two exercises determine if the set of vectors are linearly independent. For any set that are linearly dependent, find a dependence relationship among the vectors.

9. $\begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 4 \\ 4 \end{bmatrix}$

10. $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$

11. $\begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$

12. $\begin{bmatrix} -2 \\ 3 \\ 7 \end{bmatrix}$, $\begin{bmatrix} 4 \\ -1 \\ 5 \end{bmatrix}$, $\begin{bmatrix} 3 \\ 1 \\ 3 \end{bmatrix}$, $\begin{bmatrix} 5 \\ 0 \\ 2 \end{bmatrix}$

13. Let $A = \begin{bmatrix} 3 & 0 \\ -1 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 4 & -2 & 1 \\ 0 & 2 & 3 \end{bmatrix}$,$C = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$, $D = \begin{bmatrix} 0 & -3 \\ -2 & 1 \end{bmatrix}$

$E = \begin{bmatrix} 4 & 2 \end{bmatrix}$, $F = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$. Compute the indicated matrices (if possible).

$A + 2D$, AB, $B - C$, BD, BF, EB.

14. Given an example of a nonzero 2×2 matrix A such that $AA = O$. (O is zero matrix)

15. Let $A = \begin{bmatrix} 2 & 1 \\ 6 & 3 \end{bmatrix}$. Find 2×2 matrices B and C such that $AB = AC$ but $B \neq C$.