DENTAL IMAGING USING SPARSE ANGLE X-RAY CT WITH TIKHONOV REGULARIZATION

A. Meaney[1] and P. Rantala[2]

[1] Division of Materials Physics, Department of Physics, University of Helsinki, Finland.
[2] Division of Atmospheric Sciences, Department of Physics, University of Helsinki, Finland.

alexander.meaney@helsinki.fi
pekka.a.rantala@helsinki.fi

INTRODUCTION

X-ray computed tomography (CT) is an inverse problem where the aim is to obtain a cross-sectional image of an object from its X-ray projections. The object is imaged from several different directions, and its internal structure is recovered by using a *reconstruction algorithm*.

One of the main development directions of CT imaging is reducing the radiation dose received by the patient. A straightforward way of achieving this is to reduce the number X-ray projections taken. The standard reconstruction algorithm in CT, *filtered backprojection* (FBP), works well when using many projections taken at small angular intervals. However, in *sparse tomography*, where only a few projections are used, FBP fails and the need arises to use *algebraic* and *iterative* reconstruction algorithms.

In this study, we imaged the removed wisdom tooth of a 23-year-old female using sparse tomography. Reconstructions were computed using classical *Tikhonov regularization* (TR) with two different methods for determining the regularization parameter α. Our aim was to test the feasibility of this approach to reconstruction by comparing the results to an FBP reconstruction obtained from densely imaged data.

MEASUREMENTS AND METHODS

We considered the linear matrix model $\mathbf{m} = A\mathbf{f} + \boldsymbol{\epsilon}$, where A is the tomographic measurement matrix, $\mathbf{m}$ is the measured data, $\mathbf{f}$ is the target function, and $\boldsymbol{\epsilon}$ models noise. This tomographic measurement model is shown in Figure 1. In Tikhonov regularization, the reconstruction $\mathbf{T}_\alpha$ is obtained by minimizing the expression $\|A\mathbf{T}_\alpha - \mathbf{m}\|^2 + \alpha\|\mathbf{T}_\alpha\|^2$ [1, p. 63].

The reconstructions were computed using a matrix-free iterative algorithm called the conjugate gradient method [2, p. 7]. The iteration was stopped when the relative difference between consecutive reconstructions fell below 0.001 %.

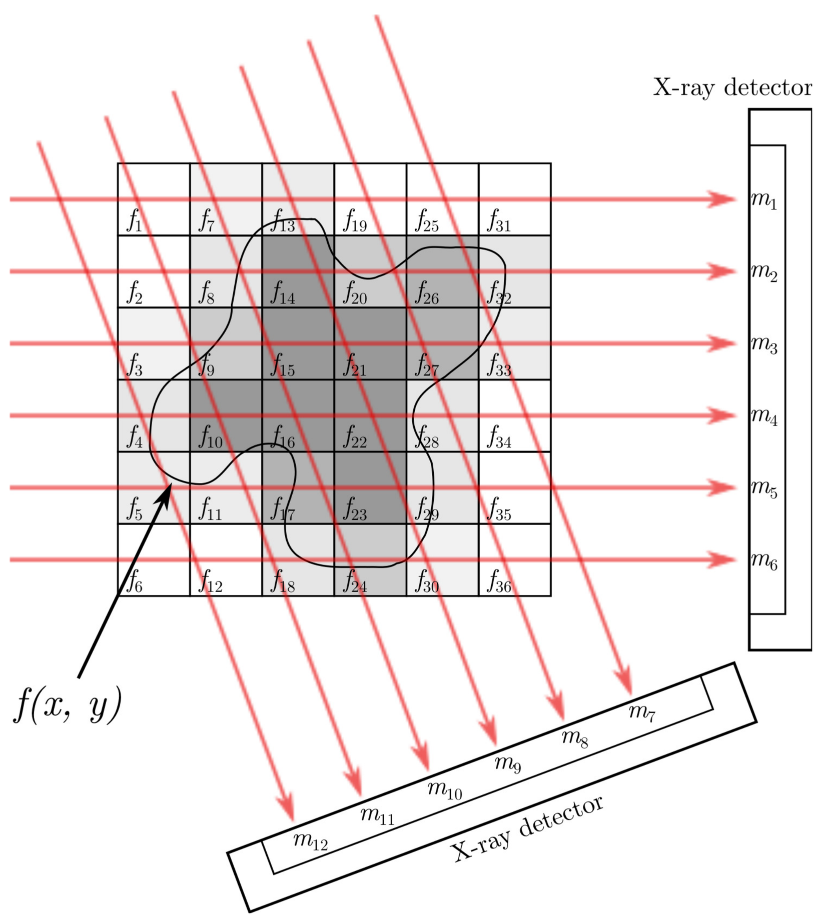

Figure 1 : The discretized tomographic model for a 6 × 6 pixel grid ($n = 36$).

We used two different approaches for determining α: Morozov's discrepancy principle and the L-curve method [1, p. 72-76]. In the Morozov approach α is found as the unique zero of the monotonously increasing function $f(\alpha) = \|A\mathbf{T}_\alpha - \mathbf{m}\| - \|\boldsymbol{\epsilon}\|$. The sinogram noise level $\|\boldsymbol{\epsilon}\|$ was estimated using the method proposed by Wang et al. [3]. In the L-curve method α is found by plotting $(\ln\|A\mathbf{T}_\alpha - \mathbf{m}\|, \ln\|\mathbf{T}_\alpha\|)$ and choosing α from the corner of the curve typically resembling the letter L.

The wisdom tooth was imaged in the Industrial Mathematics CT Laboratory. Two slices, one from the root and the other from the crown, were reconstructed using 10 projections. For comparison, FBP reconstructions were computed using 10 projections for a sparse data reconstruction and 180 projections for the ground truth. The reconstructions were morphologically segmented by greyscale thresholding, preceded by median-filtering. The relative cross-sectional areas of the different tissues, dentine and enamel, were computed and compared to the ground truth [4].

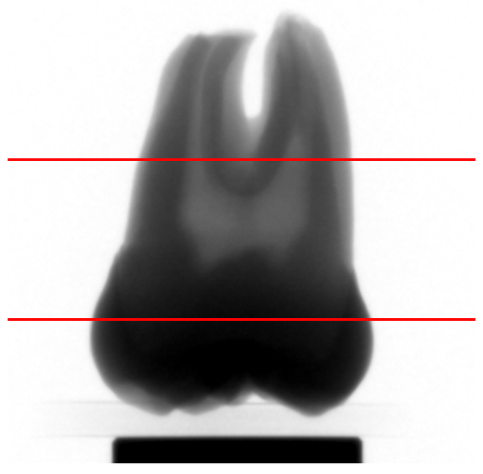

Figure 2 : The reconstruction planes in the dental root (upper line) and in the dental crown (lower line).

RESULTS

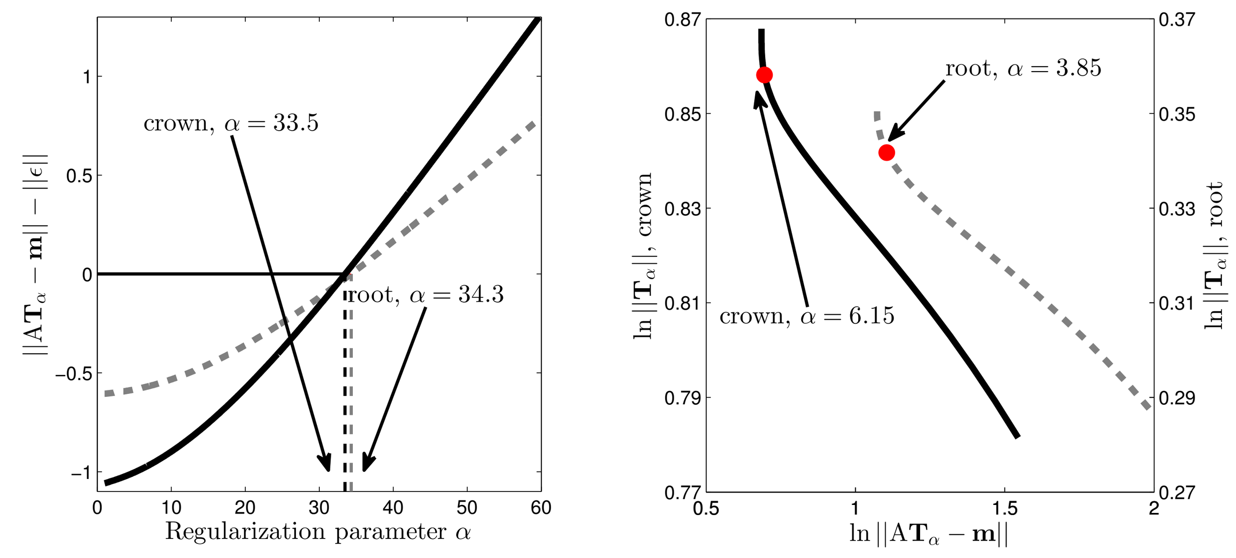

Figure 3 : Determining the regularization parameter using Morozov's discrepancy principle (*left*) and the L-curve method (*right*).

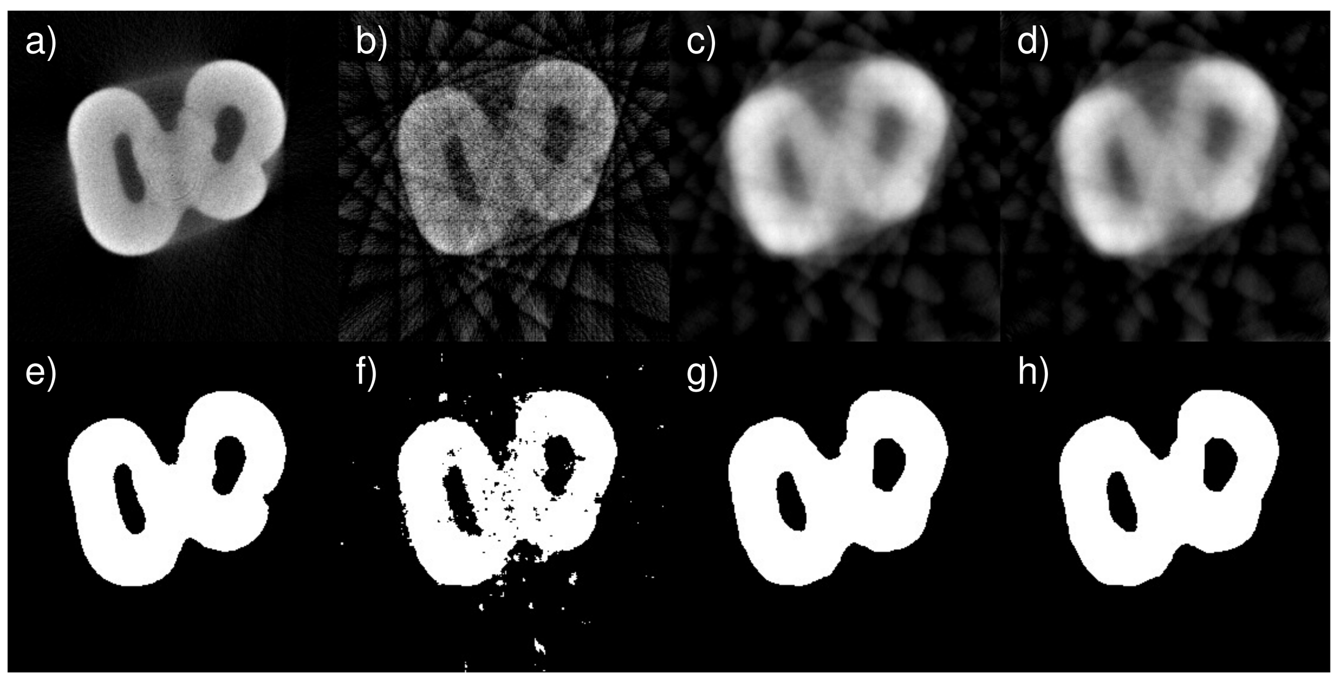

Figure 4 : Root reconstructions with segmentation into dentine and air. Computed using FBP, sparse angle FBP, TR with the Morozov principle, and TR with the L-curve method (Fig. a-e, b-f, c-g, and d-h, respectively).

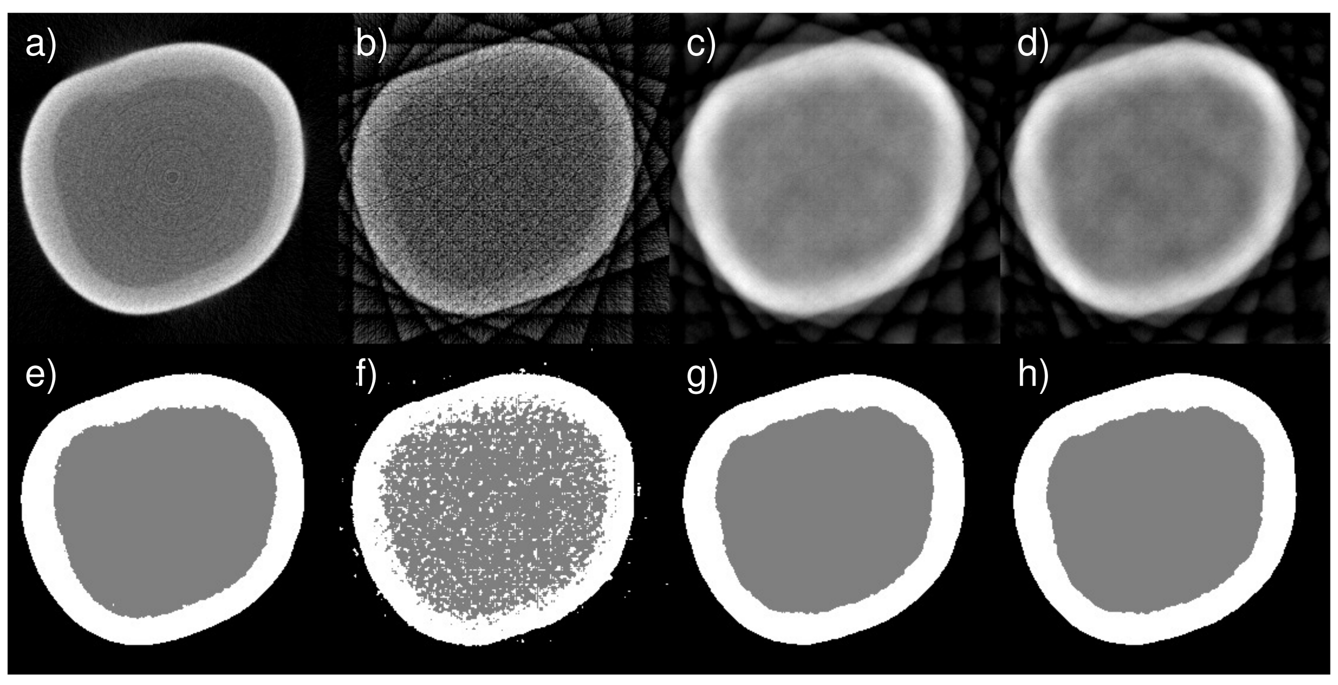

Figure 5 : Crown reconstructions with segmentation into dentine (grey), enamel (white) and air. Computed using FBP, sparse angle FBP, TR with the Morozov principle, and TR with the L-curve method (Fig. a-e, b-f, c-g, and d-h, respectively).

Table 1 : Cross-sectional areas of dentine in the dental root, and enamel and dentine in the dental crown as determined from the different reconstructions.

	Dental root		Dental crown		
Method	α	Dentine	α	Dentine	Enamel
FBP (ground truth)	-	100 %	-	100 %	100 %
FBP (sparse)	-	101.2 %	-	90.2 %	116.4 %
Tikhonov, Morozov	34.3	102.2 %	33.5	96.7 %	102.3 %
Tikhonov, L-curve	3.85	102.9 %	6.15	96.6 %	103.9 %

CONCLUSIONS

- Tikhonov regularization gave smoother reconstructions and more reliable segmentation results compared to the sparse angle FBP.
- The regularization parameter α could be determined using both the Morozov principle and the L-curve method. Although the values differed somewhat, both approaches resulted in very similar reconstructions.
- TR seemed to work better in the crown than in the root. This was most likely due to the strong beam hardening effects in the latter.

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HELSINGIN YLIOPISTO
HELSINGFORS UNIVERSITET
UNIVERSITY OF HELSINKI
MATEMAATTIS-LUONNONTIETEELLINEN TIEDEKUNTA
MATEMATISK-NATURVETENSKAPLIGA FAKULTETEN
FACULTY OF SCIENCE