Fractal sets in analysis, Exercise 5, 25.2.2015

1. Prove that the Hausdorff metric d_H really is a metric.

2. Is it true that if $K_i, i = 1, 2, \dots$, and K are compact subsets of $\mathbb{R}$ and $d_H(K_i, K) \to 0$, as $i \to \infty$, then $\dim K_i \to \dim K$? If not, could it be that always $\dim K \leq \liminf_{i\to\infty} \dim K_i$?

3. Prove that the metric space $(\mathcal{K}(\mathbb{R}^n), d_H)$ is separable, that is, it has a countable dense subset.
Hint: Study finite subsets.

4. Prove that the metric space $(\mathcal{P}(Q), d_L)$ is separable, where Q is the unit cube $\{x \in \mathbb{R}^n : 0 \leq x_i \leq 1 \ \forall i = 1, \dots, n\}$.

5. Prove that the mapping F on page 16 of the lecture notes is a contraction.

6. Prove that the mapping M on page 16 of the lecture notes is a contraction.