Fractal sets in analysis, Exercise 4, 18.2.2015

1. Prove that for every Besicovitch set $B \subset \mathbb{R}^n$, $\dim B \geq 2$. Hint: You may use that this is true when $n = 2$.

2. Prove that if $B_1 \subset \mathbb{R}^m$ and $B_2 \subset \mathbb{R}^n$ are Besicovitch sets, then $B_1 \times B_2 \subset \mathbb{R}^{m+n}$ is a Besicovitch set.

3. Show that if for all n every Besicovitch set in $\mathbb{R}^n$ has Hausdorff dimension at least $n-1$, then for all n every Besicovitch set in $\mathbb{R}^n$ has upper Minkowski dimension n.
Hint: Consider the product sets $B^k = B \times \cdots \times B$ for large k and use exercises 3.1 and 3.2.

4. Write down explicit linear maps giving the Sierpinski gasket as the attractor. Do the same for the von Koch curve.

5. Prove that for a self-similar set $K = \cup_{j=1}^N f_j(K)$ the open set condition is satisfied if all the different parts $f_j(K)$ are disjoint.