**Fractal sets in analysis, Exercise 2, 4.2.2015**

1. Show that $\dim A \leq \dim_P A \leq \overline{\dim}_M A$.

2. Prove the formulas

$$\underline{\dim}_M A = \liminf_{\delta \to 0} \frac{\log N(A, \delta)}{\log(1/\delta)},$$

and

$$\overline{\dim}_M A = \limsup_{\delta \to 0} \frac{\log N(A, \delta)}{\log(1/\delta)}.$$

3. Show that if $F$ is a compact AD-regular set, then $\dim F = \dim_P F = \underline{\dim}_M F = \overline{\dim}_M F$.

4. Show that if $F = \{0, 1, 1/2, 1/3, \dots\}$, then $\underline{\dim}_M F = \overline{\dim}_M F = 1/2$.

5. Prove (essentially the easier part of Frostman's lemma) that if $A \subset \mathbb{R}^n$ and $\mu$ is a Borel measure such that $\mu(A) > 0$ and

$$\mu(B(x, r)) \leq r^s \quad \text{for all } x \in \mathbb{R}^n, r > 0,$$

then $\mathcal{H}^s(A) > 0$.