Department of Mathematics and Statistics, University of Helsinki
Numerical methods and the C language, Winter and Spring 2014

Workshop 10
Mon 31.3. at 16-18 B322 FILE: h10.tex printed on January 11, 2014 at 16.19.

1. Ramanujan has proposed that

$$\int_0^\infty \frac{dx}{(1+x^2)(1+r^2x^2)(1+r^4x^2)} = \frac{\pi(1+r+r^2)}{2(1+r^2)(1+r)^2}.$$

   By using the program `xqromo.cpp` or QAGI-integration in GSL show that this formula holds for $r = 0, 1, \ldots, 10$ and compute the size of the error.

2. Solve the following by using the program `mysix4.cpp` (or in GSL `simplex.c`):

   a) Maximize $x_1 + 2x_2 + 3x_3 - x_4$ subject to $x_i \geq 0$ and

$$x_1 + 2x_2 + 3x_3 \leq 15$$

$$2x_1 + x_2 + 5x_3 \leq 20$$

$$x_1 + 2x_2 + x_3 + x_4 \leq 10$$

   b) Maximize $2x_1 - 4x_2 + 5x_3 - 6x_4$ subject to $x_i \geq 0$ and

$$x_1 + 4x_2 - 2x_3 + 8x_4 \leq 2$$

$$-x_1 + 2x_2 + 3x_3 + 4x_4 \leq 1$$

3. Minimize

$$5x^2 + \sin(5x) - 2\cos(x^2)$$

   by using the golden section and Brent's method (NR::brent, NR::golden). Compare their rate of convergence, i.e. print or plot the x-values and function values after each iteration step.

   Hint: Modify NR::brent and NR::golden to obtain the x-values after each step of the algorithm.

4. The Bessel function J_0 has the following approximate formula

$$\pi J_0(z) = \left(\frac{2\pi}{z}\right)^{1/2} \left\{\cos(z-\pi/4) + \frac{1}{8z}\sin(z-\pi/4) + O(1/z^2)\right\} .$$

   Explore experimentally the maximal error in this formula on the intervals $[10*j, 10*(j+1)], j = 1, ..., 30$. Recall that `NR::bessj0` gives the values of this Bessel function in NR.

5. A designer wants to find a spline curve that nicely describes a horizontal cross section of the famous Aalto vase. Help the designer in this task.

   Hint: Use the program `mysptrp2.cpp`. See the picture below.