Department of Mathematics and Statistics, University of Helsinki
Numerical methods and the C language, Winter and Spring 2014

Workshop 8
Mon 17.3. at 16-18 B322 FILE: h08.tex printed on February 17, 2014 at 19.32.

1. Consider a linear system $Gz = w$ where G is an $n \times n$ matrix with complex coefficients and w is an $n \times 1$ vector. Write these as $G = A + i * C$, $z = x + i * y$, and $w = b + i * d$ where A, C, x, y, b, d have real entries. If we consider the real and imaginary parts separately we can reduce the system (see p.53 of NRC++) to the solution of a real $2n \times 2n$ system

```
A x - C y = b
C x + A y = d.
```

   Write a program that

   (a) generates the coefficients A, C, b, d,

   (b) solves the aforementioned real $2n \times 2n$ system for x and y

   (c) carries out the complex multiplication $(A + i * C) * (x + i * y)$ and compares it to $b + i * d$ and prints the norm of the difference vector.

2. According to a theorem of Hadamard $|\det(A)| \leq n^{n/2}$ whenever A is an $n \times n$ matrix with entries in $[-1, 1]$. Verify this experimentally when $n = 3$ and also for $n = 4$ completing in both cases the following steps:

   (a) Generate 1000 random matrices as above, and for each matrix A compute $|\det(A)|$ and plot a histogram of the values.

   (b) Check that the stated result holds.

3. The program seigen on the www-page should compute, for a symmetric real matrix, the eigenvectors (column j of eigevec returns the j th eigenvector) and the eigenvalues.

   (a) Test the accuracy of this program for a symmetric random matrix, by computing for each eigenvalue λ_i and each eigenvector x_i the expression $|a * x_i - \lambda_i * x_i|$ in the following format:

```
i        lambda_i       |A*x_i -lambda_i*x_i|
  0      5.75344976        6.916654e-16
  1      1.63485437        6.425880e-16
  .....
```

   (b) If a is a $m \times n$ matrix, then $a^T a$ is $n \times n$. With seigen, show that the positive square roots of the eigenvalues of $a^T a$ are the singular values of a. Hint. You may want to modify the program ShowUWV/mycond.cpp from the www-page.

4. The program mysteep5.cpp is given on the course www-page.

a) In the main program, a portion of the code is commented out. Modify the program by releasing the code from the comment and record the least function value found by the original program and the modified program in 10 tests.

b) Repeat the comparison, but starting the algorithm at the points $(0.4 * m, 0.4 * n), m, n = -3, ..., 5$, and for each initial point write the least function value found by both the original and the modified program in a file. Use the following format

```
x   y   f1      f2
 .......
```

where f1 refers to the value by the original program and f2 to the modified program.

5. Show by experiments that the all the expressions below have the same value and determine this value, so called Catalan's constant.

(a)
$$\frac{1}{2}\int_0^1 \mathcal{K}(x)dx, \;\; \mathcal{K}(x) = \frac{\pi}{2} {}_2F_1(\frac{1}{2}, \frac{1}{2}; 1; x^2).$$

(b)
$$-\frac{1}{2} + \int_0^1 \mathcal{E}(x)dx, \;\; \mathcal{E}(x) = \frac{\pi}{2} {}_2F_1(-\frac{1}{2}, \frac{1}{2}; 1; x^2).$$

(c)
$$\int_0^1 \frac{\arctan(x)}{x}dx.$$

Hint: It is probably best to use the representations for $\mathcal{K}(x)$ as `NR::ellf(0.5*M_PI,x)` and for $\mathcal{E}(x)$ as `NR::elle(0.5*M_PI,x)`.