Department of Mathematics and Statistics, University of Helsinki
Numerical methods and the C language, Winter and Spring 2014

Workshop 6

Mon 24.2. at 16-18 B322 FILE: `h06.tex` printed on February 17, 2014 at 14.08.

1. Suppose that A is a real symmetric square matrix. Recall that such a matrix is positive definite, if all of its eigenvalues are positive.

   (a) Write a function `IsPosDefS` which returns 1 if a is a symmetric real positive definite matrix and 0 otherwise.

   (b) Let A be a symmetric real $n \times n$ matrix and I the identity matrix of the same size. Show that there exists $c \in \mathbb{R}$ such that the eigenvalues of $cI + A$ are positive (prove first that the eigenvalues of $cI + A$ are $c + \lambda_i$ where λ_i are the eigenvalues of A). Write a function `MakePosDef` which uses this idea to transform A to a positive definite matrix.

   (c) The Hadamard product (A, B) of two $m \times n$ matrices A, B is the $m \times n$ matrix with entries $a_{ij}b_{ij}$. Show that if A and B are positive definite, then so is their Hadamard product.

2. The following linear systems of equations have coefficient matrices with determinant equal to 0. Hence they cannot be solved in the usual sense, but they still can be solved in the LSQ sense by finding the pseudoinverses of the coefficient matrices and the LSQ solutions. Find the LSQ solutions.

   (a)
   ```
   x + y + z =1
   x + y + z =2
   x + y + z =3
   ```

   (b)
   ```
    x + 2y + 3z = 1
   4x + 5y + 6z = 2
   7x + 8y + 9z = 3
   ```

3. Modify the program `mytemp4.cpp`, which fits the quadratic function

   $$g(x, y) = ax^2 + by^2 + cxy + dx + ey + f$$

   to given data and prints the coefficients a, b, c, d, e, f, such that the initial data points are the same eight cities as in the comment at the end of the file `mytemp4.cpp`, but with temperatures given in the table. Also print the estimated temperatures at 5 other locations, not included in the initial data, and compare the computed values to the real values obtained from the table. Note that in order to draw the border lines, `mytemp4.cpp` needs the files `bordfin.dat` and `bordala.dat`, (Borders of continental Finland, and the Åland islands) which can be found as comments at the end of the program itself.

4. An astronomer has the following observations about a comet approaching the Earth. Determine the equation of the comet on the basis of this data using a quadratic function

   $$ay^2 + bxy + cx + dy + e = x^2.$$

Table 1: Comet coordinates

x	1.02	0.95	0.87	0.77	0.67	0.56	0.44	0.30	0.16	0.01
y	0.39	0.32	0.27	0.22	0.18	0.15	0.13	0.12	0.13	0.15

5. Write a program that plots the graph of the functions $f(x) = a_1 \sin(b_1 x) + a_2 \sin(b_2 x)$, $f'(x)$, and $g(x) = \int_0^x f(t)dt$ where the coefficients are random numbers.

   Hint: Use `numder` for differentiation and either the trapez rule or the Simpson rule for the numerical integration.

Coordinate table for exercise 3: Temperatures for some cities 11.2.2002, at 11 am

City	x	y	Temp.
Helsinki	4.6	0.6	1.3
Hämeenlinna	4.3	1.7	-0.2
Ivalo	6.0	12.3	-6.2
Joensuu	7.6	3.9	0.2
Jyväskylä	5.3	3.4	0.5
Kajaani	6.6	6.1	-1.2
Kilpisjärvi	3.2	12.8	-4.6
Kittilä	4.9	10.9	-6.0
Kokkola	3.7	5.7	-0.4
Kotka	6.0	1.0	0.9
Kouvola	5.9	1.4	0.0
Kristiinankaupunki	2.5	3.7	0.6
Kuopio	6.4	4.2	0.5
Kuusamo	7.3	8.4	-6.3
Lahti	5.2	1.6	0.2
Lappeenranta	6.8	1.9	0.3
Maarianhamina	1.0	0.8	0.8
Mikkeli	6.3	2.7	1.2
Muonio	4.3	11.1	-4.0
Nurmes	7.3	5.3	0.0
Oulu	5.1	7.1	-0.4
Pietarsaari	3.4	5.5	-0.1
Pori	2.7	2.5	1.3
Porvoo	5.3	0.7	0.9
Rovaniemi	5.4	9.3	-5.9
Savonlinna	7.3	2.8	0.7
Seinäjoki	3.4	4.3	0.2
Sodankylä	5.7	10.5	-6.3
Tammisaari	3.6	0.3	1.3
Tampere	3.9	2.5	0.3
Tornio	4.5	8.3	-1.6
Turku	2.8	0.8	-0.1
Utsjoki	6.0	14.0	-5.2
Uusikaupunki	2.4	1.5	0.8
Vaasa	2.7	4.7	-0.2
Vantaa	4.6	0.7	0.2
Varkaus	6.6	3.4	0.8