Department of Mathematics and Statistics, University of Helsinki
Numerical methods and the C language, Winter and Spring 2014

Workshop 5

Mon 17.2. at 16-18 B322 FILE: `h05.tex` printed on December 31, 2013 at 10.41.

1. Interpolate the sine-function in the interval $[0, \pi/2]$ using the points

$$x_k = k * \pi/10, \quad y_k = \sin(x_k), \quad k = 0, 1, \ldots, 5.$$

Use the following different methods:

(a) polynomial interpolation,

(b) cubic spline interpolation, with $f'(0) = f'(\pi/2)$ as boundary condition (in GSL "periodic boundary condition").

(c) cubic spline interpolation with natural boundary condition.

In each case, compute the maximum error at the points $z_j = k * \pi/200, k = 0, 1, \ldots, 100$.

2. Given $f(1) = 1,\ f(2) = 125,\ f(3) = 729,\ f(4) = 2197$, find a polynomial approximation for the numbers $f(2 + i * 0.25),\ i = 0, 1, 2, 3, 4$, using the program `polint` or `gpolint` (`polint` is a NR function, `gpolint` can be found at the webpage). Find also the corresponding error estimates. Do the same thing for the data $(i, f[i]),\ i = 1, 2, 3, 4$, where `f[i]` is a random number.

3. Compute the values of the Γ function at the points $x = 0.5 + 0.1 * j, j = 1, \ldots, 20$, and form a spline approximation on this interval. Then estimate the maximal deviation of the approximation from the true value of the function by tabulating the difference with increment 0.01 on this interval.

4. Using the programs `numder` and `numdf` (or `gnumder` and `gnumdf`) and Gnuplot, make a program which draws a picture of a function and its second derivative. Draw also the sum of the function $\sin(x)$ and its second derivative, which is supposed to be zero.

5. Let A be an $m \times n, m \geq n$, matrix. Using random matrices, test the validity of the following statements for the pseudoinverse A^+ of A:

(a) $(A^+)^+ = A$,

(b) $(A^+)^\top = (A^\top)^+$

(c) If B is a $n \times p$ matrix, then $(AB)^+ = B^+A^+$.

6. Fitting a second degree polynomial

$$p(t) = x_0 + x_1 * t + x_2 * t^2$$

to the data set $(1, 2), (2, 2), (3, 3), (3, 5), (4, 6)$ leads to the overdetermined system $Ax = b$ with

$$A = [1, 1, 1; 1, 2, 4; 1, 3, 9; 1, 3, 9; 1, 4, 16] \qquad b = [2; 2; 3; 5; 6]$$

Find the coefficients of the polynomial (a) with the SVDsolve (b) with HOUSEsolve method.
Note: HOUSEsolve can be downloaded from the course www-page. It is part of the file `myhouse.c(pp)`.