Department of Mathematics and Statistics, University of Helsinki
Numerical methods and the C language, Winter and Spring 2014

Workshop 3

Mon 3.2. at 16-18 B322 FILE: h03.tex printed on December 31, 2013 at 10.39.

1. Make functions which generate random upper and lower triangular matrices and functions which solve an upper and lower triangular system of equations, $Ux = b$ and $Lx = b$ respectively. These solvers (`usolve` and `lsolve`) should take as an argument an upper or respectively a lower triangular matrix as well a constant vector b. Solve the systems for random matrices and for a randomly generated vector b.

2. For random numbers $c_0, \ldots, c_4$ and fixed $a, b \in \mathbb{R}, a < b$, compute the value of the integral
$$I(a,b) = \int_a^b \sum_{j=0}^{4} c_j x^j \, dx$$
(a) analytically, (b) numerically with the following trapezoidal formula. Let $f(x) = \sum_{j=0}^{4} c_j x^j$, $n \in \mathbb{N}$, $h = (b-a)/n$ and $x_k = a + kh, k = 0, ..., n$. Then
$$I(a,b) \approx I(a,b,n) = h \sum_{k=1}^{n} \left[\frac{1}{2} f(x_{k-1}) + \frac{1}{2} f(x_k)\right] = h \sum_{k=0}^{n} f(x_k) - \frac{h}{2}(f(x_0) + f(x_n)).$$
Print the results for $n = 10, 100, \ldots$ in the form

```
n      I(a,b,n)   |I(a,b,n) - I(a,b)|     h*h

  10      ...             ....              ...
 100      ...             ....              ...
1000      ...             ....              ...     .
```

3. Is the diagonal dominance of a square matrix preserved under the multiplication of two such matrices? Is the inverse of a diagonally dominating matrix diagonally dominating? Is the inverse of a tridiagonal matrix tridiagonal? Remember that a square $n \times n$ matrix $A = (a_{ij})$ is diagonally dominating if $|a_{i,i}| > \sum_{j=1, j \neq i}^{n} |a_{i,j}|$ for all $i = 1, \ldots, n$ and tridiagonal if $a_{i,j} = 0$ for $|i - j| > 1$.

4. At the youthful age of 103 years L. Vietoris (1891-2002) proved in 1994 the following result (Notices of AMS Nov. 2002).

Theorem. Let $a_0 \geq a_1 \geq ... \geq a_n > 0$. If $a_{2k} \leq \frac{2k-1}{2k} a_{2k-1}$ for $1 \leq k \leq \frac{n}{2}$, then for all $t \in (0, \pi)$
$$f_1(t) \equiv \sum_{k=1}^{n} a_k \sin kt > 0\,, \text{ and } f_2(t) \equiv \sum_{k=0}^{n} a_k \cos kt > 0.$$
Verify these inequalities by generating random sequences of the coefficients satisfying these constraints and by graphing the functions f_1, f_2.

5. For real $n \times n$ matrices A with eigenvalues λ_i show that the following results hold

$$\mathrm{tr}(A) \equiv \sum_{i=1}^{n} a_{i,i} = \sum_{i=1}^{n} \lambda_i, \qquad \det(A) = \prod_{i=1}^{n} \lambda_i.$$

Use the program `myeigen.cpp` (www-page/Chapter 11) to verify this experimentally. If you are using GSL, it is sufficient to study symmetric matrices only.