Department of Mathematics and Statistics, University of Helsinki
Numerical methods and the C language, Winter and Spring 2014

Workshop 2

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The helping program for the exercise 5 is at the course www-page.

1. Use the Monte Carlo method to compute the area between the curves $y = \sin(x)$ and $y = \cos(x)$ in the rectangle $\{(x, y) : 0 < x < 2\pi, -1 < y < 1\}$.

2. The LSQ solution of the linear system $Ax = b$ where A is $m \times n$, $m > n$, is given according to Linear Algebra I by $x = (A^T A)^{-1} A^T b$ if $(A^T A)^{-1}$ exists. Use this to fit the LSQ line $y = x_1 + x_2 t$ to the data $\{(2, 1), (5, 2), (7, 3), (8, 3)\}$.

3. Suppose that P and R are invertible $p \times p$ and $r \times r$ matrices, resp., and let
$$A = \begin{pmatrix} P & 0 \\ 0 & R \end{pmatrix}, B = \begin{pmatrix} P^{-1} & 0 \\ 0 & R^{-1} \end{pmatrix}$$
Show (e.g. by experiments) that $B = A^{-1}$.

4. (a) Make a function which takes a matrix as an argument and zeros all its entries below the diagonal. Write then a program that uses the function and `ranmat` to generates a random upper triangular matrix. Is it true that the product / inverse of an upper triangular matrix is again triangular?

   (b) Make a function which takes a matrix $A = (a_{ij})$ as an argument and makes all $a_{i,j} = 0$ for $|i - j| > 1$. Write then a program using the function and `ranmat` to generate a random tridiagonal matrix. Is it true that the product / inverse of a tridiagonal matrix is again tridiagonal?

5. Let $f(x) = \int_0^x \sin^2(t)dt$. Use the inverse function algorithm (see the www-page) to find a x such that $f(x) = 5$.