Department of Mathematics and Statistics, University of Helsinki
Numerical methods and the C language, Winter and Spring 2014

Workshop 1

Mon 20.1. at 16-18 B322 FILE: h01.tex printed on December 31, 2013 at 13.55.

1. The formula to calculate a Celsius wind chill is:

$$T(wc) = 0.045(5.27V^{0.5} + 10.45 - 0.28V)(T - 33) + 33$$

Where: $T(wc)$ = the wind chill, V = the wind speed in kilometers per hour and, T = the temperature in degrees Celsius. Write a program to compute the wind chill. *Hint.* Use the program `h1p011.c(pp)` on the www-page as a starting point.

2. Use the function in problem 1 to print the values of wind chill factor for the wind speeds $2 * j m/s, j = 0, 1, 2, 3, 4$ and temperatures $10 - j * 5, j = 0, 1, 2, 3, 4$ in the following format

```
0   10  5  0  -5 -10
2   ....
4   ....
6   ....
8   ....
```

Hint. You may compare the results with a table the www-page h012.eps.

3. The file h013.dat on the www-page contains 21 (x, y)-pairs, one pair per line. Use this data to numerically approximate dy/dx and write the approximations, 20 $(x, y'(x))$-pairs, on the screen or into a file.

4. The following table gives the euro exchange rate in US dollars at 6 consecutive Mondays. Use this information to fit a least-squares line $ax + b = y$ to the data $(x_i, y_i), i = 1, \ldots, 6$, where $x_i = i$ is the ordinal of the given date and y_i the corresponding exchange rate. Use vectors to store the data.

Table 1: Average exchange rates, 2001

Date	22.10.	29.10.	5.11.	12.11.	19.11.	26.11.
1 EUR in USD	0.8969	0.9005	0.8961	0.8919	0.8793	0.8818

Hint: Generally, for $(x_i, y_i), i = 1, \ldots, n$, the formulas of the coefficients a and b are

$$a = \frac{\sum x_i y_i - \frac{1}{n} \sum x_i \sum y_i}{\sum (x_i - \bar{x})^2}, \quad b = \frac{\sum y_i - a \sum x_i}{n},$$

where $\bar{x} = \frac{1}{n} \sum x_i$ is the mean value.

5. Use the fixed point iteration to solve the equations (a) $\cos(x) = x$, (b) $e^{-x} = x$, (c) $1 - \cosh(x) = x$.

6. The arithmetic-geometric mean $\mathrm{ag}(a, b)$ of two positive numbers $a > b > 0$ is defined as $\mathrm{ag}(a, b) = \lim a_n$, where $a_0 = a, b_0 = b$, and

$$a_{n+1} = (a_n + b_n)/2, \quad b_{n+1} = \sqrt{a_n b_n}, \quad n = 0, 1, 2, \ldots$$

(a) Write a function, which takes two arguments (`double`), computes ag and returns the value (`double`).

(b) The hypergeometric function ${}_2F_1(a, b; c; x)$ is defined as a sum of the series,

$$\begin{aligned} {}_2F_1(a, b; c; x) = 1 + \frac{ab}{c}\frac{x}{1!} + \frac{a(a+1)b(b+1)}{c(c+1)}\frac{x^2}{2!} + \ldots \\ + \frac{a(a+1)\ldots(a+j-1)b(b+1)\ldots(b+j-1)}{c(c+1)\ldots(c+j-1)}\frac{x^j}{j!} + \ldots \quad . \end{aligned}$$

This hypergeometric series converges for $|x| < 1$. Gauss proved in 1799 that there is a connection between the hypergeometric function and the arithmetic-geometric mean,

$${}_2F_1(\tfrac{1}{2}, \tfrac{1}{2}; 1; r^2) = \frac{1}{\mathrm{ag}(1, \sqrt{1 - r^2})}$$

for $0 < r < 1$. Tabulate the difference of the two sides of this identity for $r = 0.05k, k = 1, \ldots, 19$. Use a library routine to calculate the values of the ${}_2F_1$.