Department of Mathematics and Statistics
Measure and Integral
Exercise 7
Extra exercises, 2014

You may gain extra credit points by returning your solutions (in written) either to the lecturer or your instructor by Monday, March 3, 4 pm.

1. Find the limit
$$\lim_{t\to 0}\int_1^\infty \frac{\cos(tx)}{1+x^2}\,dx.$$

2. Let $A \in \operatorname{Leb}\mathbb{R}^n$, $m(A) < \infty$, and let (f_j) be a sequence of integrable functions $f_j\colon A \to \dot{\mathbb{R}}$. Suppose that $f_j \to f$ uniformly in A. Prove that f is integrable and
$$\lim_{j\to\infty}\int_A f_j = \int_A f.$$

3. Let $f\colon \mathbb{R} \to \mathbb{R}$ be integrable and $a \in \mathbb{R}$. Prove that the function $F\colon \mathbb{R} \to \mathbb{R}$,
$$F(x) = \int_{-\infty}^x f(t)\,dt,$$
is continuous.

4. Let $f\colon \mathbb{R}^n \to \mathbb{R}$ be integrable. Prove that for every $\varepsilon > 0$ there exists a measurable $A \subset \mathbb{R}^n$ such that $m(A) < \infty$ and
$$\int_{\mathbb{R}^n\setminus A} |f| < \varepsilon.$$

5. Find the limit
$$\lim_{n\to\infty}\int_{-n}^n e^{-nx^2}\,dx.$$

6. Find the limit
$$\lim_{k\to\infty}\int_0^\infty \frac{\sin(x^k)}{x^{k-1}}\,dx.$$