# Outline

History

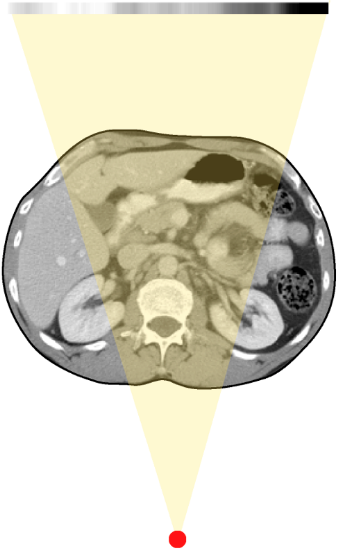
Modelling X-ray attenuation

Matrix model approach to tomography

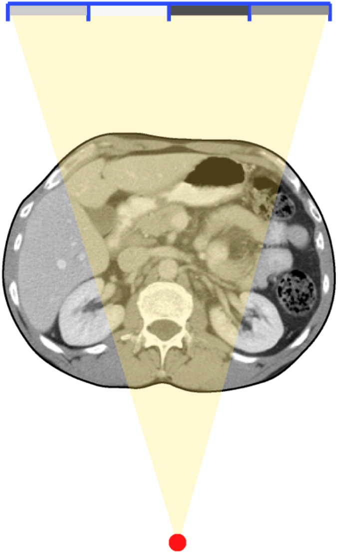
Continuous vs. discrete tomography

Visibility and invisibility

# X-ray tomography: Continuum model



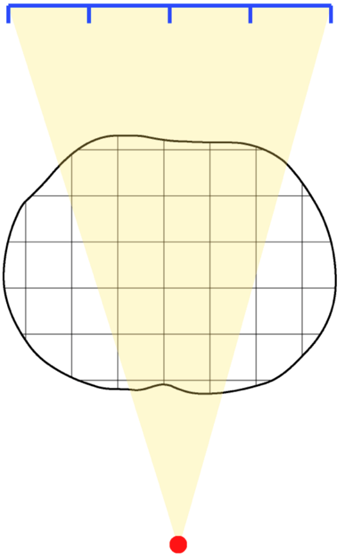
# X-ray tomography: Practical measurement model



$m \in \mathbb{R}^k$

$k = 4$

# X-ray tomography: Computational model



$m = Ax$

$m \in \mathbb{R}^k$

$x \in \mathbb{R}^n$

$k = 4$

$n = 48$

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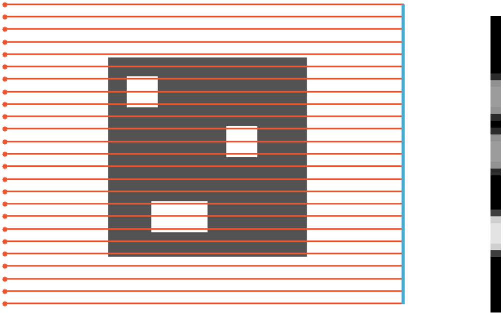
# Construction of the sinogram



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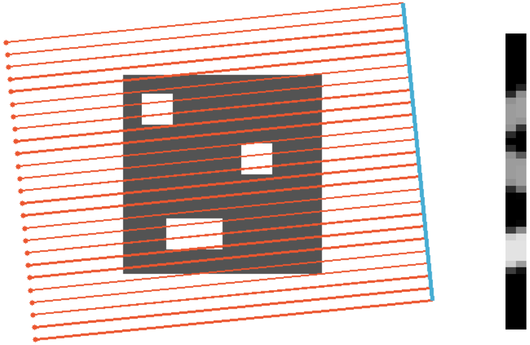

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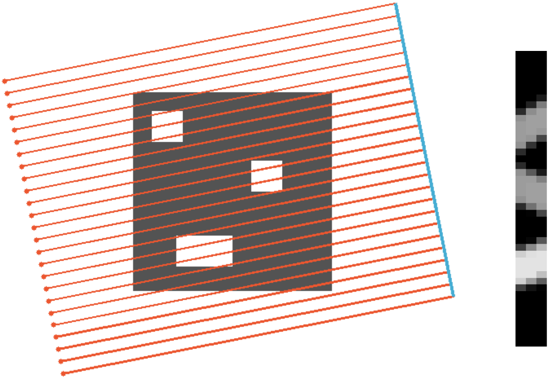

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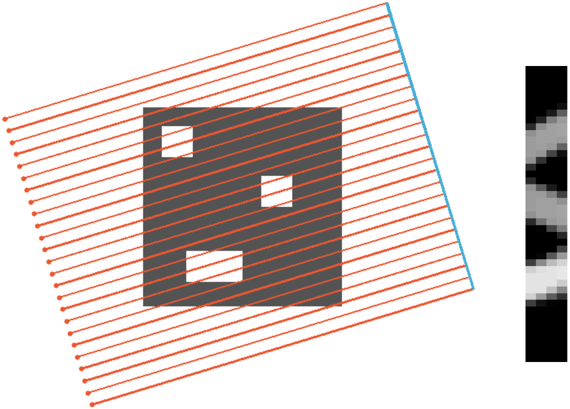

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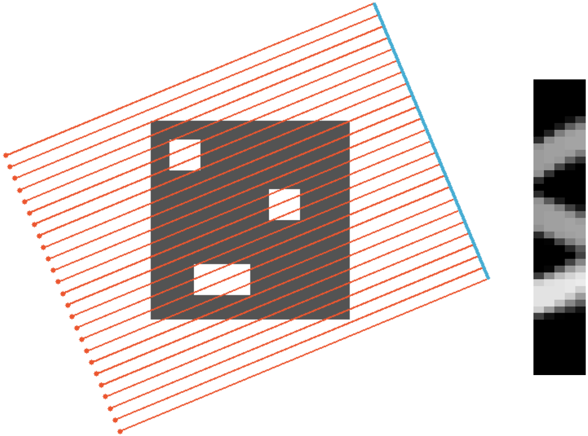

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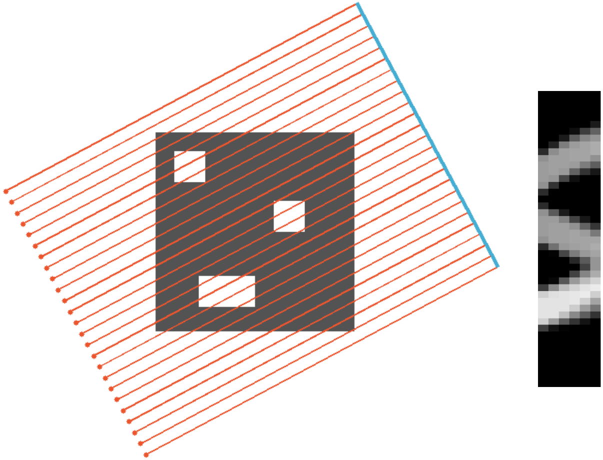

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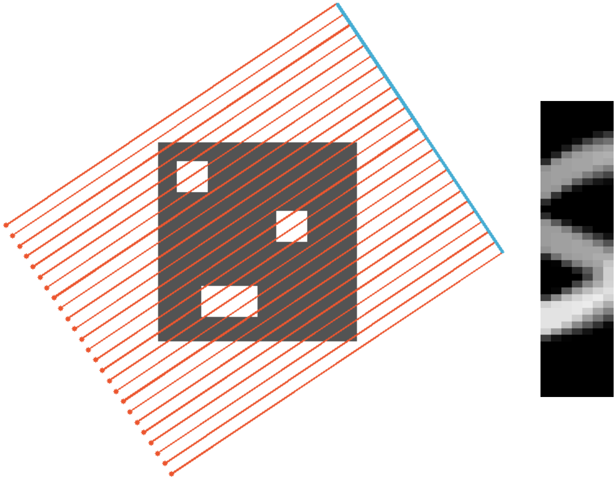

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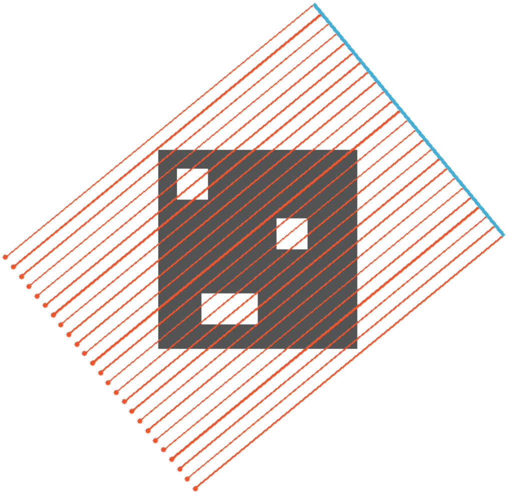

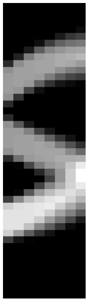

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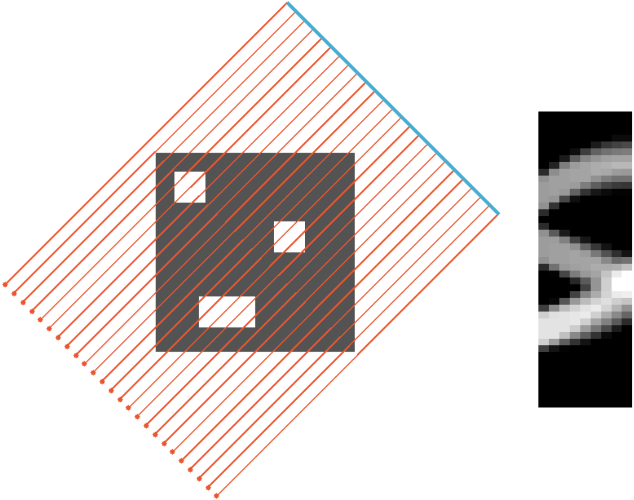

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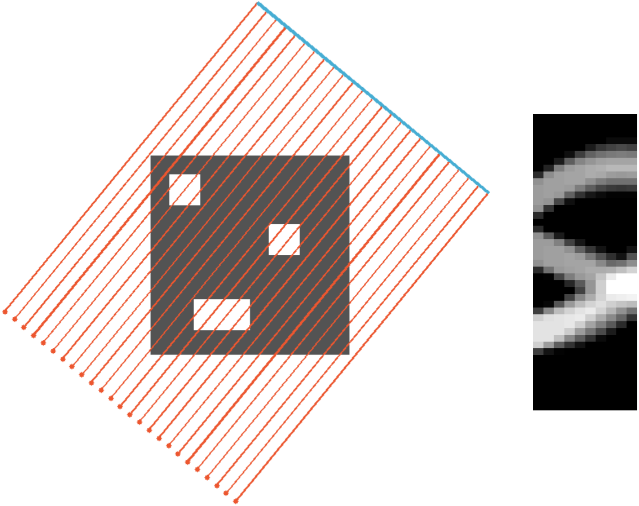

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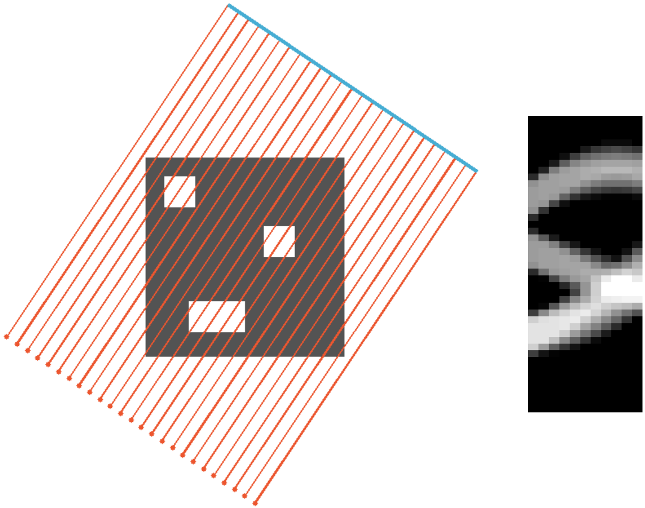

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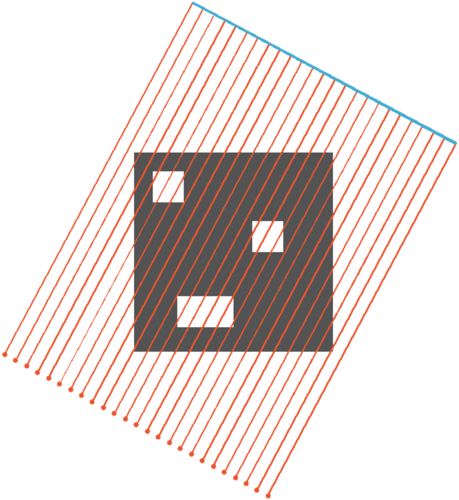

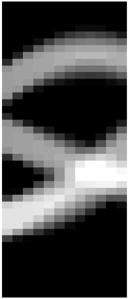

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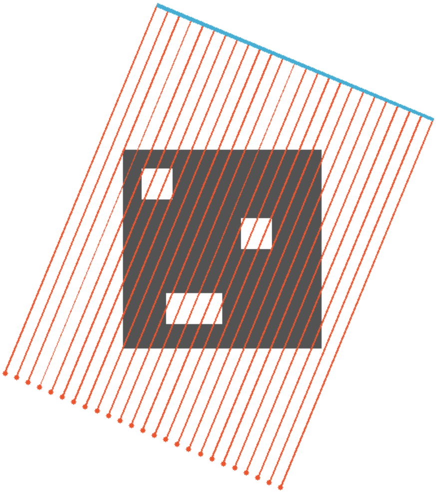

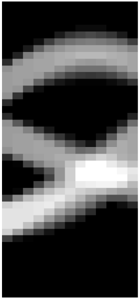

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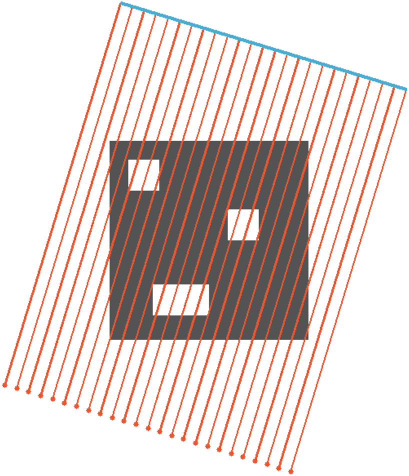

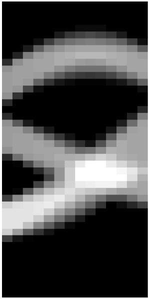

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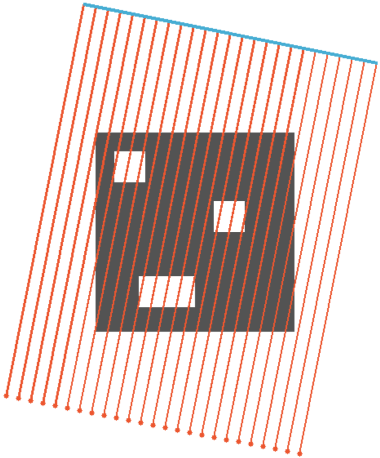

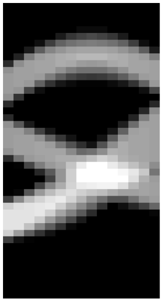

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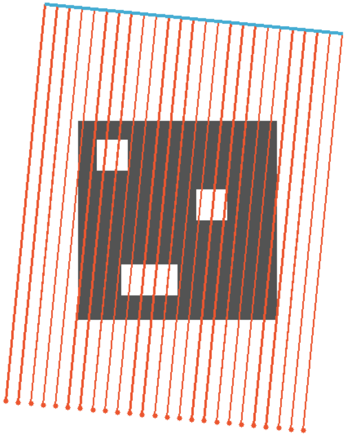

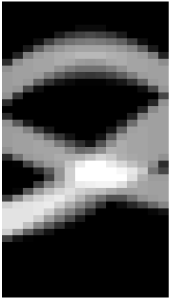

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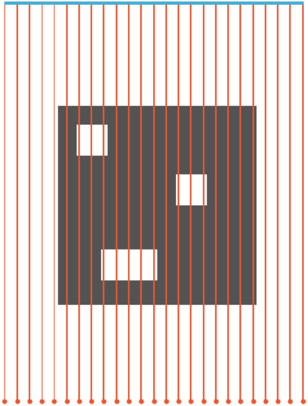

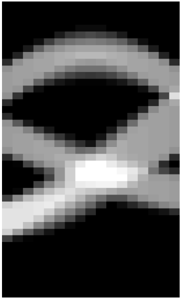

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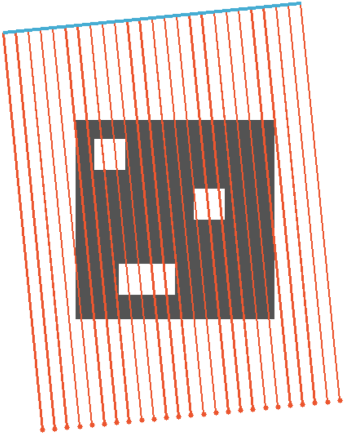

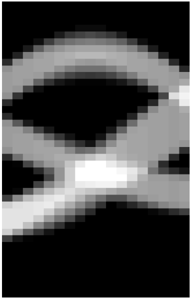

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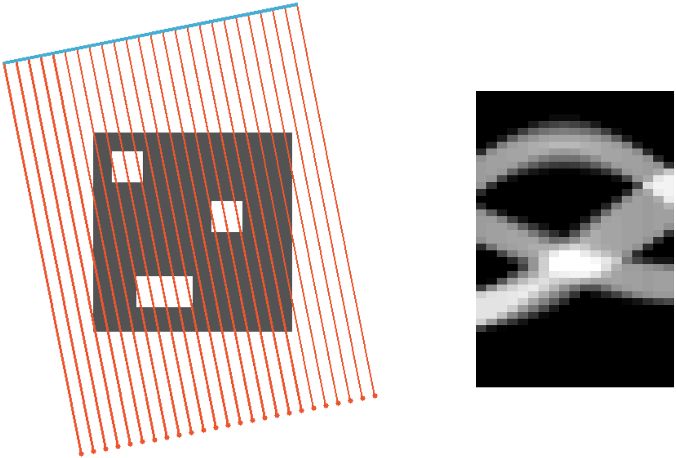

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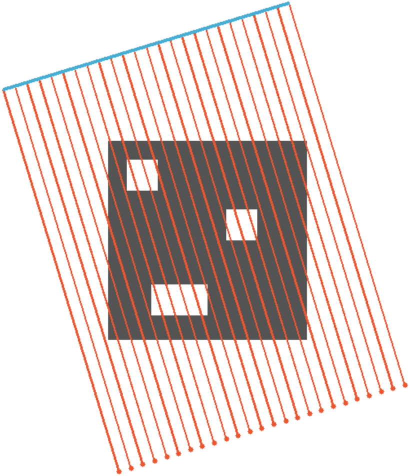

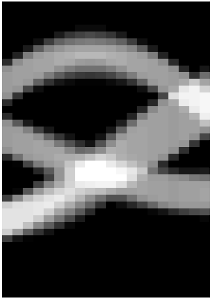

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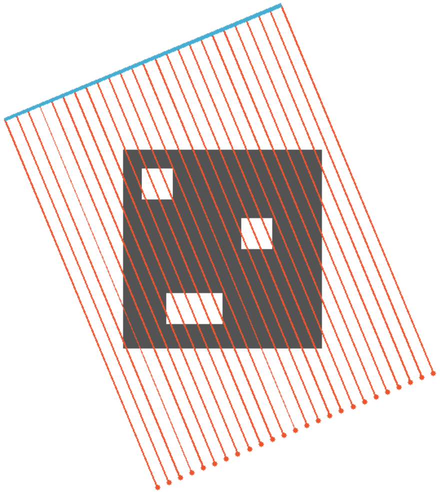

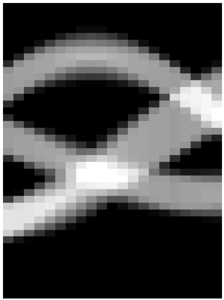

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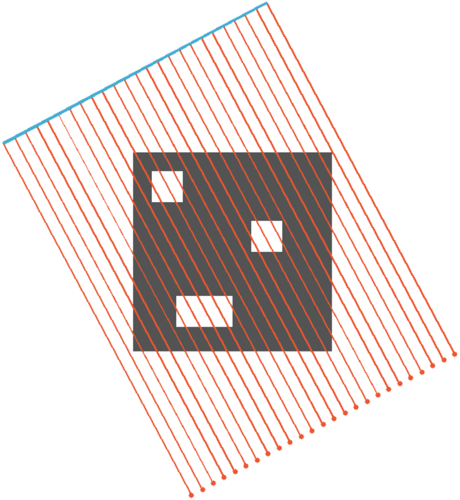

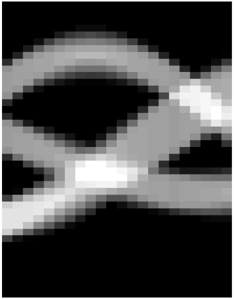

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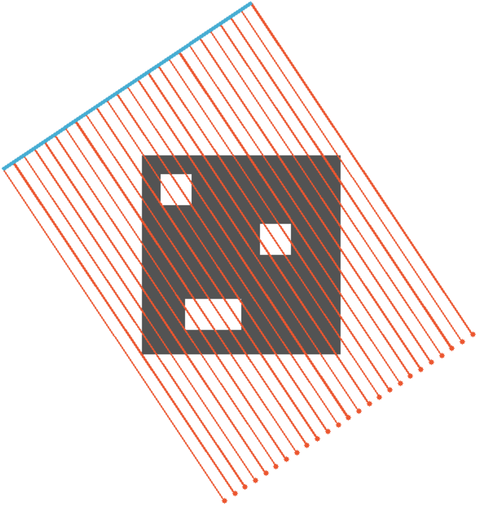

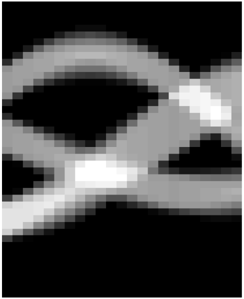

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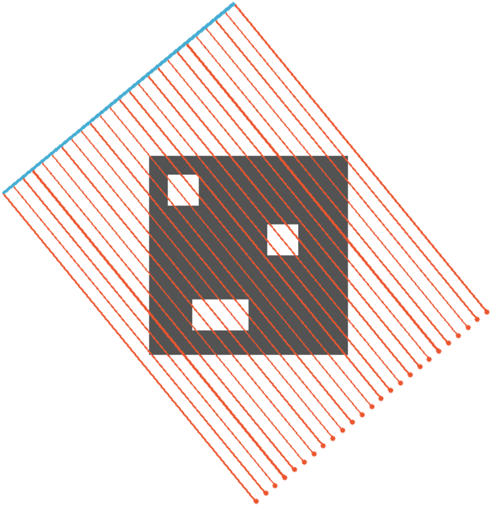

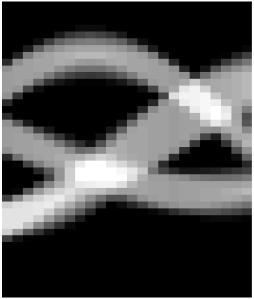

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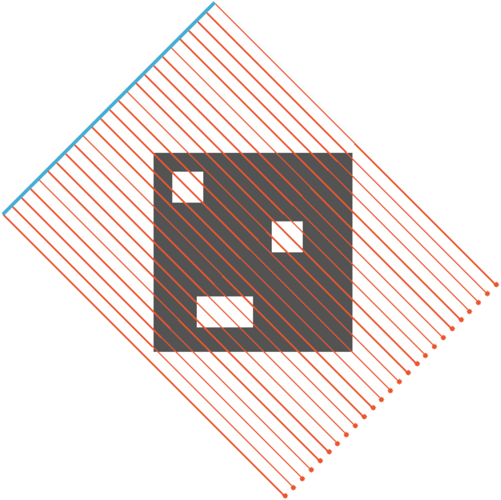

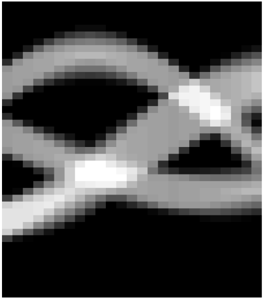

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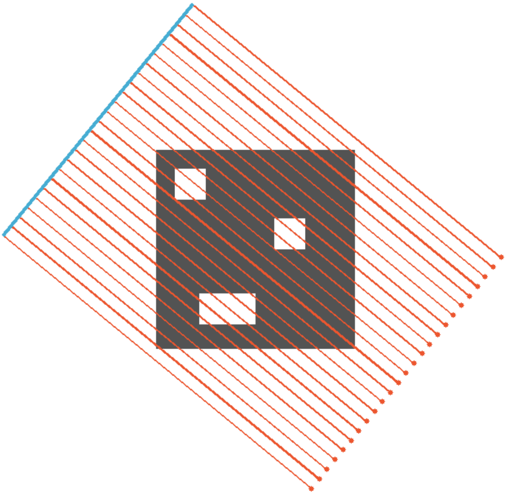

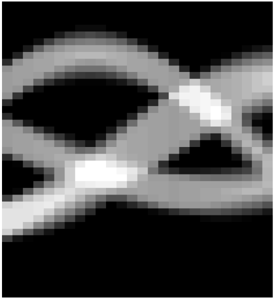

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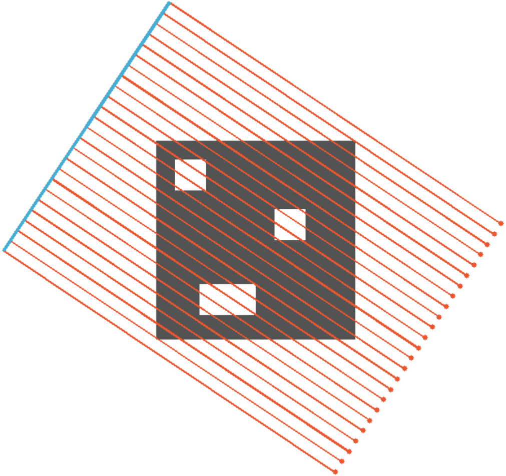

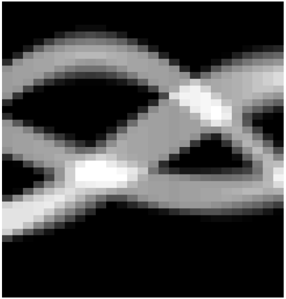

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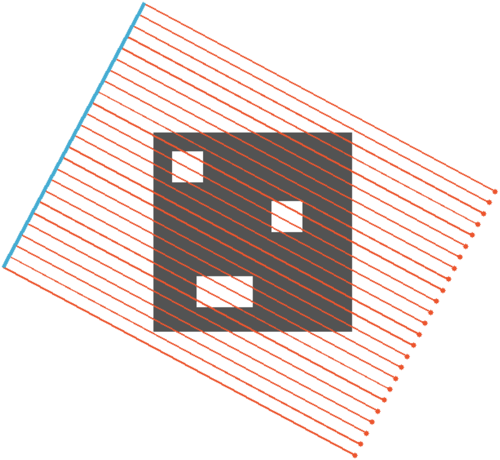

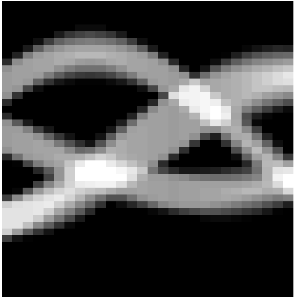

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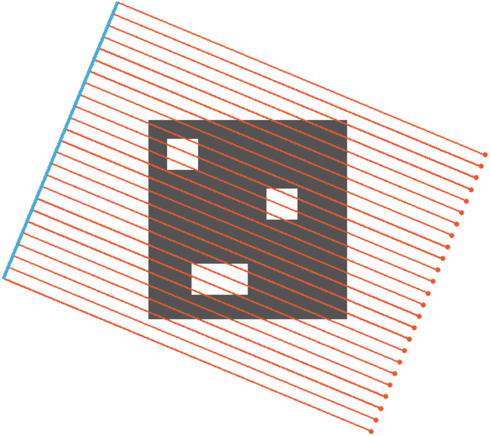

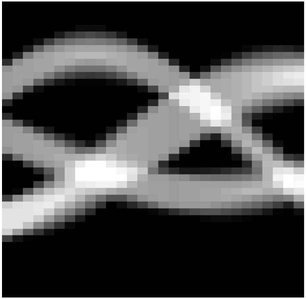

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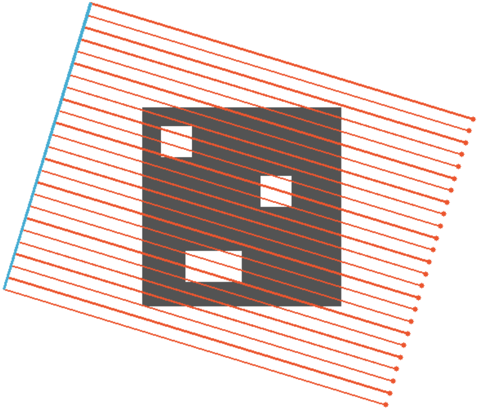

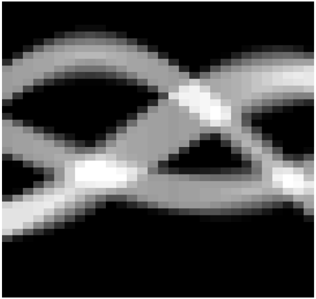

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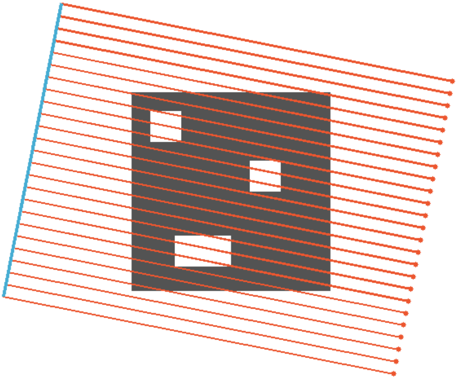

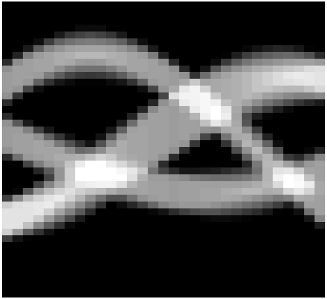

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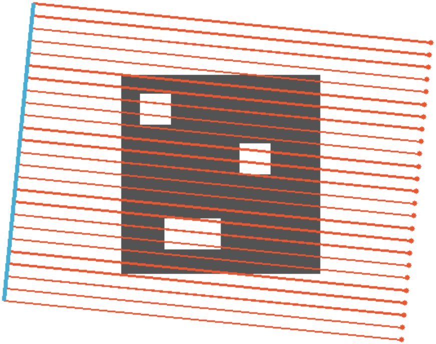

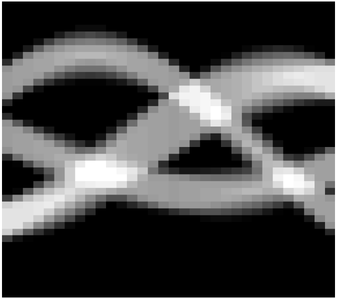

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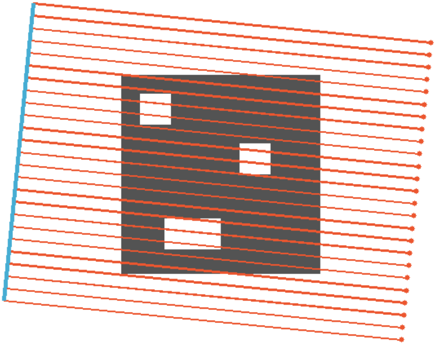
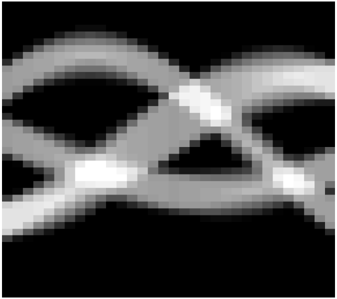
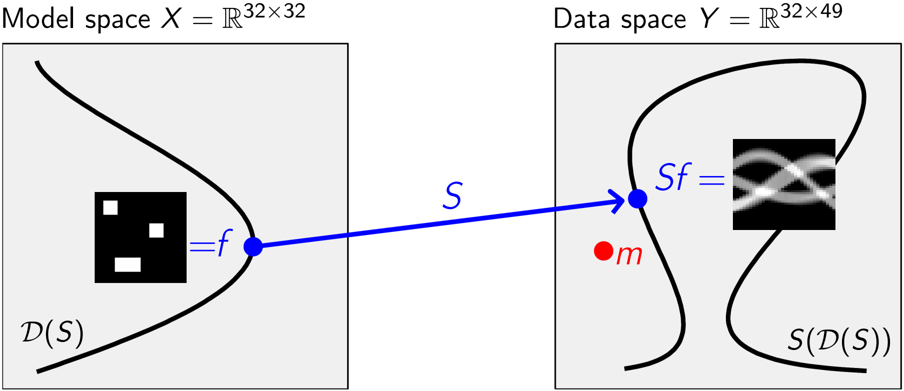




Unknown: $f \in \mathbb{R}^{32\times32}$

Data: $Sf \in \mathbb{R}^{49\times32}$

## Inverse problem of X-ray tomography: given noisy sinogram, find a stable approximation to $f$




Consider the measurement model $m = Sf + \varepsilon$. We want to know $f$, but all we can do is measure $m$ that depends indirectly on $f$. Moreover, the measurement is corrupted with additive noise $\varepsilon$.

# Nonuniqueness in X-ray tomography

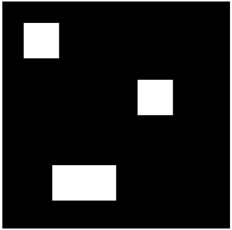
It was noted already in [Cormack 1963], and later analyzed in [Smith, Solmon & Wagner 1977, Theorem 4.2], that a finite number of line integrals does not determine the target uniquely since the measurement operator has a nontrivial nullspace.

> THEOREM 4.2. *A finite set of radiographs tells nothing at all.*
>
> For some reason this theorem provokes merriment. It is so plainly one of those mathematical ideals untainted by any possibility of practical application.

However, the *ghosts*, or the objects invisible in the tomographic data, are high-frequency functions and can be effectively suppressed by regularization.

# These phantoms have almost the same sinogram

Original phantom



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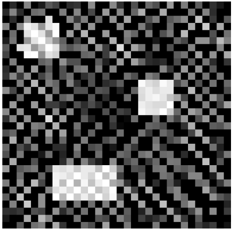

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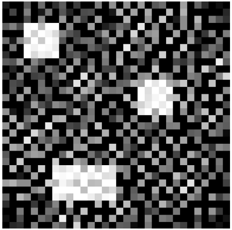

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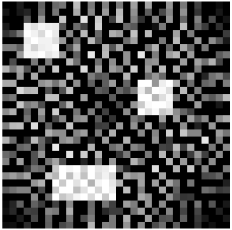

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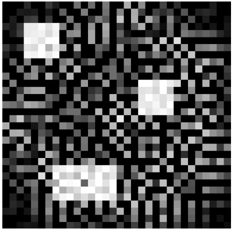

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